The Optimal Share of Property Rights Protection Expenditure in Total Government Spending: The Case of Iran

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Abstract
The main purpose of this paper is to obtain the optimal amount of expenses of government relating to the protection of property rights (PPRs). To achieve these purposes we have introduced concept of social intelligence with respect to PPRs and then developed different growth model from existing literature. In the second step the optimal share of government spending on the PPRs is calculated. The theoretical results show inverse relationship between budget deficits and government spending in the PPRs. In other words, with increasing amount of government deficit, government reduced spending of PPRs. The results of calibration for Iranian economy show that, the growth rate of spending to PPRs should be equal to 31 percent for having sustainable economic growth rate of 6 percent.

Keywords: Augmented Endogenous Growth Model, Property Rights, Calibration, Iran.
JEL classification: O43, B52, E11.

1. Introduction
Many studies introduce the issue of incomplete protection of property rights (PPRs) in economic growth models. In these studies, it has been
endeavoured to formulate the different nature of the incomplete PPRs. A considerable part of the existing studies is considered with the spending related to the incomplete PPRs such as reduction in investment due to the issue of predation and political rent seeking. The studies of Tornell and Velasco (1992), Tornell and Lane (1999), Grossman and Kim (1996), Lindner and Strulik (2004), Mino (2006) and Gonzalez (2007) are some of these studies. Svensson (1998), Gradstein (2004) and Dincer and Ellis (2005) studied the effects of incomplete PPRs on growth and accumulation of capital within the framework of overlapping generations models. Svensson (1998) has tried to determine the optimal amount of expense on choosing the efficient level of legal system (or the equilibrium level of investment in legal infrastructure) and Dincer and Ellis (2005) assume that the rate of PPRs is a proportion of production which can be supported by the firms. The studies of Teng (2000) and Sylwester (2001) were carried out within the framework of games theory. Teng (2000) has set a two stage game with two players, ‘government’ and ‘large number of economic agent’; and Sylwester (2001) has also made a two stage game, in which it has been assumed that entrepreneurs are dispossessed of a part of their production. In some studies such as Palda (1999), Grossman and Kim (1996) and Anderson and Bandiera (2005) some methods have been applied, which are different from the Ramsey models of optimization, overlapping generations model and the game theory. Palda (1999) measures the condition of property rights with a parameter which is a portion of the existing income that is there to be confiscated by rent seekers. Palda (1999) relates the optimal level of property rights to the number of rent seekers. In all studies it has been assumed that there is no complete PPRs. In all studies, the number of interest groups or the decision of individuals to enter productive or non-productive activities (e.g. rent seeking, predation, etc.) have often been applied as indicators of a lack of a full PPRs. Rennani et al. (2008) have assessed the effect of spending on property rights protection on economic growth and welfare, within the framework of Ramsey type growth models. The issue that makes this study different from the others is that in this study, the spending on
PPRs has been applied and it has been directly introduced in growth model. Also its effect on economic growth has been analyzed. There is no study in which the optimal government spending share on PPRs, within endogenous growth models, has been clarified. The purpose of this article is to fill this gap, and it will be studied theoretically in the next section. Accordingly, the theoretical contributions of this paper can be seen as follows:

1. Defining and describing a concept called ‘social intelligence with respect to property rights’.
2. Driving the relationship between social intelligence with respect to PPRs, and the growth rate of government expenditures in PPRs sector.
3. Driving the relationship between the social intelligence and government expenses on PPRs.
4. Driving the relationship between government spending on PPRs and discount rate.
5. Driving the relationship between government spending in PPRs and government budget.
6. Calculating the optimal amounts of government spending in the PPRs, and also calculating the growth rate and the share of expenditure in PPRs from the total government spending.

The rest of the paper is organized as follows. In the second section a developed model has been offered. In the third section the solution of the model has been inserted. The fourth section presents the numerical solution of the constructed model based on the data from Iranian economy. The fifth section deals with calibrating the model and section six concludes.

2. Base Model
The main purpose of this paper is to calculate the optimal amount of spending related to PPRs in total government spending, and also to obtain the growth rate and its share of the total spending, and to analyze their optimal path within framework of an endogenous growth model. To do so, following legal-centralist view, we propose an augmented growth model that is different from the existing studies.

2.1 Welfare Function
Review of welfare function in previous studies showed that private
consumption is one of the important variables in welfare function. Also this variable is necessary for securing the stability condition. Paying attention to concept of intertemporal preferences will take place when individuals’ property rights are protected. Therefore it is obvious that the amount of spending related to PPRs will enter social welfare function will result in consumer’s preferences to be considered endogenous. Therefore utility function at any specific time can be written as:

\[ U(t) = \left( \frac{C(t)^{1-\theta} - 1}{1-\theta} \right) + \eta \left( \frac{G_p(t)^{1-l} - 1}{1-l} \right) \]  

(1)

Where:

- \( \eta \): The share of government services (resulting from property rights) in utility
- \( G_p \): Government spending in PPRs
- \( C \): Consumption
- \( \theta \): Inverse of intertemporal elasticity of substitution
- \( U \): Instantaneous utility
- \( 1-l \): Social intelligence with respect to PPRs (level of social sensitivity with respect to property rights)

By taking derivative from instantaneous utility function with respect to spending related to PPRs we will have:

\[ \frac{\partial U}{\partial G_p} = \eta \frac{1}{G_p^l} \]

Since \( G_p \) is greater than 1, the higher value of \( l \), the less sensitivity of utility to changes in \( G_p \) becomes. Thus we have called \( 1-l \) the level of social sensitivity or social intelligence with respect to property rights (or understanding the importance of PPRs by society). More \( 1-l \) (the rise in society understands of the importance of PPRs) means more utility sensitivity with respect to the increase in \( G_p \). Therefore, based on equation (1), social welfare function can be defined:

\[ W = \int_0^\infty U(t) e^{-r^*t} dt = \int_0^\infty \left[ \left( \frac{C(t)^{1-\theta} - 1}{1-\theta} \right) + \eta \left( \frac{G_p(t)^{1-l} - 1}{1-l} \right) \right] e^{-r^*t} dt \]  

(2)
In which $\rho$ is discount rate, and other variables are the same as before. This welfare function is different from the other studies. The noticeable point about equation (2) is that the services provided by the government are considered to come about as PPRs. This seems to be an appropriate assumption for some public services such as national security, defense and etc. Taking $(1-\ell)$ as social intelligence with respect to property rights into account is one of the differences between equation (2) and other studies, and is an important contribution of this study.

### 2.2 Equations of Motion

In this study, two equations of motion are considered: Equations of motion for physical capital and human capital. Following Rennani et al. (2008), the equation of motion for physical capital can be written as:

$$\dot{K} = (1+\xi-t)Y - \delta K - C = fY - \delta K - C$$

Where:

- $f = (1+\xi-t)$
- $t$: income tax rate
- $\xi$: Budget deficit in proportion to GDP ($Y$) or $\xi = \frac{T-G}{Y}$ which a constant. We’ll have a balanced budget when $\xi = 0$.
- $K$: the total (private and public sectors) physical capital.
- $C$: Private sector’s consumption.
- $Y$: total (private and public sectors) production, which can be defined as:

$$Y = \alpha K^{\sigma_K} H^{1-\sigma_K} G_p^{\sigma_G}$$

In equation (4) we define the amount of $\alpha$ as follows:

$$\alpha = [B(1-\chi)(1-\nu)^{\sigma_K} (1-\nu)^{1-\sigma_K} + AN^{1-\sigma_K} (\frac{\epsilon}{\eta})^{\sigma_G} G^{\sigma_G} u^{1-\sigma_K}]$$

Where:

- $A$: Transition parameter
- $\sigma_K$: Production elasticity with respect to physical capital
- $1-\sigma_K$: Production elasticity with respect to human capital
\( \sigma_G \): Production elasticity with respect to the services offered by government

\( \theta_R \): Elasticity related to capital congestion

\( e_j \): An index for the legal system’s efficiency

\( \gamma \): Share of PPRs in total government spending

\( G_p \): Government’s spending in PPRs

\( B \): Transitional parameter

\( \chi \): X-inefficiency coefficient (inefficiency of government firms)

\( u \): The share of labor in private sectors (the share of labor in public sector is equal to \( 1 - u \))

\( v \): The share of physical capital in private sector (the share of physical capital in public sector is equal to \( 1 - v \))

Rennani et al. (2008) have only considered the equation of motion for physical capital. In this article, however, the equation of motion for human capital has also been considered, as below:

\[
\hat{H} = g_h H
\]  
(5)

It has been assumed that human capital grows at a steady growth rate \( (g_h) \). Based on equations (2), (3), (5) and (5) the base model constructed in the present article can be summarized as follows:

\[
\max \int_0^\infty \left( \frac{C(t)^{1-\sigma} - 1}{1 - \theta} + \eta \frac{G_p(t)^{1-\gamma} - 1}{1 - l} \right) e^{-\rho t} dt
\]  
(6)

S.T

\[
\dot{K} = fY - \delta K - C
\]  
(3)

\[
Y = \alpha K^{\sigma_k} H^{1-\sigma_k} G_p^{\sigma_G}
\]  
(4)

\[
\dot{H} = g_h H
\]  
(5)

3. Solving and Developing the Model
In this section, we only deal with solving of optimization problem. The objective is to calculate growth rates and optimal amounts of variables in steady state. The results acquired from this section have been analyzed in the next part of the article.
3.1 Initial Conditions of Welfare Maximization

In order to solve the maximization problem, we set the current Hamiltonian Function:

$$HA = \left(\frac{C(t)^{1-\theta}}{1-\theta} + \eta \frac{G_p(t)^{1-l}}{1-l}\right) e^{-\rho t} + \lambda_1 [fY - \delta K - C] + \lambda_2 [g_p H]$$

In which $\lambda_1, \lambda_2$ are the shadow prices of physical and human capital. ‘C’ is consumption and ‘$G_p$ ’ is government expenditure on PPRs, (these are control variables). ‘K’ is physical capital, and H is the labor (these are the state variables).

From first order condition for maximization:

$$\frac{\partial HA}{\partial C} = 0 \Rightarrow C(t)^{\theta} e^{-\rho t} = \lambda_1$$

By taking the logarithm of the equation (7), and then taking differential we, will have:

$$\frac{\lambda_1}{\lambda_2} = -\theta \left( C \frac{\partial C}{C} - \rho \right)$$

The other conditions for maximization will be in the form of the equations (9), (10) and (11):

$$\frac{\partial HA}{\partial G_p} = 0 \Rightarrow \eta G_p^{1-l} e^{-\rho t} = -\lambda_1 \frac{Y}{G_p} \Rightarrow \lambda_1 = -\frac{\eta G_p^{1-l} e^{-\rho t}}{Y}$$

$$\frac{\partial HA}{\partial K} = -\lambda_1 \Rightarrow -\lambda_2 = \lambda_1 \left( f \frac{\sigma g Y}{K} - \delta \right) \Rightarrow \frac{\lambda_2}{\lambda_1} = -\left( f \frac{\sigma g Y}{K} - \delta \right)$$

$$\frac{\partial HA}{\partial H} = -\lambda_2 \Rightarrow -\lambda_2 = \lambda_1 f \left( \frac{1- \sigma g Y}{H} + \lambda_2 g_h \right)$$

In equations 9 to 11 we have applied the derivative of production function to capital$^4$.

We define the rates of consumption, production, the spending related to PPRs and accumulation of physical capital growth like bellow:
\[
\frac{\dot{C}}{C} = g_c, \quad \frac{\dot{Y}}{Y} = g_y, \quad \frac{\dot{G}_p}{G_p} = g_{G_p}, \quad \frac{\dot{K}}{K} = g_K
\]

(12)

By equalizing the equation 8 and 10 and simplification we will have:

\[-(\frac{\sigma_k Y}{K} - \delta) = -\theta \frac{\dot{C}}{C} - \rho \Rightarrow g_c = \frac{1}{\theta} (-f \frac{\sigma_k Y}{K} + \delta + \rho)\]

(13)

We can arrange the equation (13) basis on \(\frac{Y}{K}\):

\[
\frac{Y}{K} = \frac{\theta g_c + \delta + \rho}{f \sigma_k} = \text{constant}
\]

(14)

In steady state, the growth rate should be constant. Therefore in steady state we will have:

\[
\dot{g}_Y = \dot{g}_K = \dot{g}_H = \dot{g}_c = \dot{g}_{G_p} = 0
\]

(15)

3.2 Calculating the Growth Rate of Spending on PPRs Based on Optimal Growth Rate

In this section, we have calculated the long term growth rate of PPRs expenditure. By arranging the equation (9) and taken logarithm of both sides of that equation we will have:

\[
\lambda_i = -\frac{\eta G_p^{1-t} e^{-\rho t}}{f \sigma G} \Rightarrow -\frac{\eta G_p^{1-t} e^{-\rho t}}{f \sigma G} = \lambda_i Y \Rightarrow
\]

\[
\ln -\frac{\eta}{f \sigma G} + (1-t) \ln G_p - \rho t = \ln \lambda_i + \ln Y
\]

(16)

We take differential of both sides of this equation. Considering the equation (10), (16) and with some simplification, the growth rate of expenditure devoured to PPRs will be attained as:

\[
g_{G_p} = \frac{1-\theta}{1-l} g^*
\]

(17)

In section 4.1, the empirical results of equation (17) have been analyzed.

It is necessary to point out that since this model is arranged for
developing countries and we have taken $l$ to be independent and constant, the relationship between economic growth rate, and the growth rate of spending on PPRs is linear and the descending.

On the other hand, if we take derivative of equation (17) with respect to social intelligence, we will have:

$$\frac{\partial g_{G_p}^*}{\partial (1-l)} = \frac{1-\theta}{(1-l)^2} g^*$$

This equation indicates that the derivative of the growth rate of the spending on PPRs is negative. This means that, with increase of social intelligence, the growth rate of expenditure on PPRs will be reduced, and private sector’s share does in turn increase.

### 3.3 Calculation of the Optimal Amount of Expenses on PPRs

In this section, given the optimal condition of GDP, the optimal amount of the expenses on PPRs has been calculated. It is assumed that the amount of government spending in PPRs is a percentage of total government spending ($G_p = \gamma G$), therefore considering the equation (9) we have:

$$G_p = \gamma (t - \xi) Y$$

(18)

On the other hand, considering the equation 7 & 9 we have:

$$C = \eta G_p^{1-l}$$

(19)

By applying the equation 12 to 14 and doing some mathematical operation we will have:

$$C = f Y(1- \frac{\sigma K g^* + \sigma K \delta}{\theta g^* + \delta + \rho})$$

(20)

With replacing $C$ from (20) into (19), and by doing some simplification, we have:

$$G_p^* = \left[\frac{(1+\xi-t)^{1-\theta} \sigma E (1- \frac{\sigma K g^* + \sigma K \delta}{\theta g^* + \delta + \rho})}{\eta} \right]^{1/l} Y^{\frac{1-l}{l}}$$

(21)

By defining $\beta$ as:
The empirical results that were calculated in equation (21) have been analyzed in the fourth section of this study.

By considering the equation 21 and by taking derivative of government expenditure on PPRs with respect to social intelligence, we will have:

$$
\frac{\partial G_p^*}{\partial (1-l)} = - \frac{G_p^*}{(1-l)^2} \ln \left( \frac{(1+\xi-t)^{1-\theta} \sigma_G (1-\frac{\sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho})^{\frac{1}{1-\theta}}} {\eta} \right)
$$

Considering this equation, we will have two situations:

$$
\begin{align*}
Y^{*+} (1+\xi-t)^{1-\theta} \sigma_G (1-\frac{\sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho})^{-\theta} > 1 & \Rightarrow \frac{\partial G_p^*}{\partial (1-l)} < 0 \\
Y^{*+} (1+\xi-t)^{1-\theta} \sigma_G (1-\frac{\sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho})^{-\theta} < 1 & \Rightarrow \frac{\partial G_p^*}{\partial (1-l)} > 0
\end{align*}
$$

And with simplification we’ll have:

$$
\begin{align*}
Y^* > [\frac{(1+\xi-t)^{1-\theta} \sigma_G (1-\frac{\sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho})^{-\theta}} {\eta}]^{\frac{1}{1-\theta}} & \Rightarrow \frac{\partial G_p^*}{\partial (1-l)} < 0 \\
Y^* < [\frac{(1+\xi-t)^{1-\theta} \sigma_G (1-\frac{\sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho})^{-\theta}} {\eta}]^{\frac{1}{1-\theta}} & \Rightarrow \frac{\partial G_p^*}{\partial (1-l)} > 0
\end{align*}
$$

Theorem 1: Equation 23 shows that, in a situation where production is less than threshold level (optimal amount), with an increase in social intelligence with respect to PPRs, government expenses would also increase. But when production level is greater than threshold level, with an increase in social intelligence respect to property rights, government can reduce the expenses on PPRs. (See also equation 26)

Theorem 2: Moreover, one can show that there is a turning point
in the function of government spending on PPRs \( G_p^* \) with respect to social intelligence \((1-l)\). The turning point of this function can be calculated as\[^{10}\]:

\[
(1-l) = \ln[Y^{1-\theta} \left(\frac{(1+\xi-t)^{1-\theta}}{\eta} - \sigma_G \left(1 - \frac{\sigma_x g^* + \sigma_k \delta}{\theta g^* + \delta + \rho}\right)\right)]^{1/2}
\]  

(24)

### 3.4 Calculating the Optimal Share of Government Spending on PPRs in Total Spending \((\gamma^*)\)

In previous part, the growth rate of the expenses on PPRs and the amount of government spending on PPRs were calculated. In this section we calculate the share of government spending on PPRs as total government spending. This share is calculated at the optimal point and with a consideration of the optimal condition of GDP. We rewrite (18):

\[
G_p^* = \gamma(t-\xi)Y^*
\]

In this section we calculate \(\gamma^*\) for a specific amount of GDP. By applying the equation 21 and 18 and omitting \(G_p^*\) from this two equations, and performing mathematical operations, we’ll have:

\[
\gamma^* = \frac{\beta}{(t-\xi)} Y^{*\frac{1-\theta}{1-\xi}}
\]

(25)

Equation 25 indicates various amounts of government spending on PPRs, at the optimal point \((\gamma^*)\) with respect to optimal condition of GDP \((Y^*)\). According to equation 25, an increase in the amount of production \((Y^*)\) will lead to a rise in the percentage of government expenses in PPRs. By taking derivative of the equation 25 we’ll have:\[^{11}\]

\[
\frac{\partial \gamma^*}{\partial (1-l)} = \gamma^* (1-\theta) \ln Y^*
\]

(26)

Considering equation 26 we can claim that when production is lower than threshold level, with an increase in social intelligence respect to PPRs, the government expenses in PPRs sector will also
increase. But when the national income level is higher than the threshold level (optimal amount), with an increase in social intelligence respect to PPRs, the government can reduce the spending related to property rights.

We have, so far, extracted the equations. In the next section we deal with analyzing the relationship between variables and equations. In the fifth section the amounts of every variable at the steady state will be calculated.

4. Empirical Results

In this section, based on the data of Iranian economy, the empirical results of model are analyzed.\textsuperscript{12}

4.1 The Relationship between Social Intelligence and the Growth Rate of Government Spending in PPRs

In this section, we analyze the relationship between the social intelligence with respect to PPRs (1-\(l\)) and the growth rate of expenditure on PPRs (\(g_{G_p}^*\)). For this aim, we have made use of the equation (17). The amounts of parameters from Table 1 have been put in this equation, and then we draw \(g_{G_p}^*\) in Figure 1 according to (1-\(l\)).

Figure 1 shows that whenever the government feels that the public enjoys a high degree of social intelligence in property rights, it can reduce the growth rate of spending on PPRs. It means that the growth rate of government spending on PPRs is a descending function of social intelligence in property rights. In other words:

\[
\frac{\partial g_{G_p}^*}{\partial (1-l)} < 0 \quad , \quad \frac{\partial^2 g_{G_p}^*}{\partial (1-l)^2} < 0
\]
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Table 1: the amount of parameters required for assessing the relationship between social intelligence in property rights and growth rate of government spending on PPRs

<table>
<thead>
<tr>
<th>parameters</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long run economic growth rate ($g^*$)</td>
<td>0.08</td>
<td>0.06</td>
<td>Model assumption and fourth development plan¹³</td>
</tr>
<tr>
<td>Inverse of intertemporal elasticity of substitution ($\theta$)</td>
<td>0.09</td>
<td>0.09</td>
<td>Abdoli (2009)¹⁴</td>
</tr>
</tbody>
</table>

Figure 1: the relationship between social intelligence in property rights ($1-l$) and growth rate of government expenditures on PPRs ($g_{G_p}$)

4.2. An Analysis of the Relationship between Social Intelligence and the Spending on PPRs

In this section, we evaluate the relationship between social intelligence and the amount of government spending on PPRs. According to equation 21 we had:

\[ G_{p}^{*} = \left[ \frac{(1 + \xi - t)^{1-\theta} \eta \sigma_{g} - \sigma_{k} g^{*} + \sigma_{k} \delta}{\theta g^{*} + \delta + \rho} \right]^{\theta} \left[ \frac{1}{1-\theta} Y^{1-\theta} \right] \]

The amounts of parameters from Table 2 are placed in equation 21. Also regarding the explanations of the third section, the amount of GDP is set to be equal to gross national income in 2008 that is 2.89
thousand trillion Rials. After replacing the amount of parameters into equation 21, we consider the government spending on PPRs \( G_p^* \) to be a function of social intelligence \( (1-l) \). \( G_p^* = G(1-l) \). Figure 2 shows \( G_p^* \) based on \( (1-l) \).

### Table 2: the amount of the parameters required for analyzing the relationship between social intelligence and the spending on PPRs

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long run economic growth ( (g^*) )</td>
<td>0.06</td>
<td>Assumption of model</td>
</tr>
<tr>
<td>Inverse of intertemporal elasticity of substitution ( (\theta) )</td>
<td>0.09</td>
<td>Abdoli (2009)</td>
</tr>
<tr>
<td>Discount rate ( (\rho) )</td>
<td>0.2</td>
<td>Is considered to be equal to interest rate</td>
</tr>
<tr>
<td>Firm’s output elasticity respect to capital ( (\sigma_k) )</td>
<td>0.42</td>
<td>Central bank of Islamic Republic of Iran (2006)</td>
</tr>
<tr>
<td>Firm’s output elasticity respect to receive of government services ( (\sigma_G) )</td>
<td>0.3</td>
<td>Rennani and et al. (2008)</td>
</tr>
<tr>
<td>Depreciation rate ( (\delta) )</td>
<td>0.1</td>
<td>It is assume that physical capital depreciation within 10 years.(^{15})</td>
</tr>
<tr>
<td>Share of government deficit in GDP ( (\xi) )</td>
<td>-0.03</td>
<td>Authors calculations based on the data of central bank 2007.(^{16})</td>
</tr>
<tr>
<td>Income tax rate ( (t) )</td>
<td>0.066</td>
<td>Authors calculations based on the data of central bank 2007.(^{17})</td>
</tr>
<tr>
<td>Social intelligence respect to property rights ( (1-l) )</td>
<td>0.5</td>
<td>Regarding to Iran is developing country.(^{18})</td>
</tr>
</tbody>
</table>

Considering Figure 2, with an increase in social intelligence, the spending on PPRs will also increase. According to equation 21, and as indicated in Figure 2, in relation to social intelligence the government expenses for PPRs \( G_p^* \) has a inflection point. This point is calculated in equation (24). This does literally mean that when social intelligence of property rights is low; the growth rate of the government spending on PPRs grows at an accelerating rate.

In contrast, when social intelligence of property rights is at a high level, growth rate of government spending on PPRs increase on
decreasing rate. Thus, if we show the social intelligence with \( \xi = 1 - I \), we’ll come to equation 27 and 28:

\[
\frac{\partial g_{G^*}}{\partial \xi} > 0 \tag{27}
\]

\[
\frac{\partial^2 g_{G^*}}{\partial \xi^2} > 0 \text{ (For a condition below threshold level)} \tag{28}
\]

\[
\frac{\partial^2 g_{G^*}}{\partial \xi^2} < 0 \text{ (For a condition above threshold level)}
\]

Figure (2): the relationship between social intelligence of property rights \((1 - I)\) and the spending on PPRs \((G_p^*)\)

4.3 An Analysis of the Relationship between Spending on PPRs and Budget Deficit
Since we had taken the theory of legal centralism into consideration, we expect that with an increase in budget deficit, the government reduces the expenses relative to PPRs, and with an increase in budget surplus it increases those expenses. Now we insert the amounts of parameters from Table 2 in equation 21. Given the amounts of parameters replaced in equation 21, the amount of government expenses for PPRs \((G_p^*)\) will be presented in the form of a function of the budget deficit \((\xi)\). Figure 3 shows the graph of the function
From Figure 3 we can conclude that with an increase in budget deficit, the government reduces the expenses on PPRs, and an increase in budget surplus leads to an increase in those expenses. This issue can be related to pro-cyclical treatment of fiscal policies in developing countries, such as Iran.

In recession period, the governments in developing countries start to reduce their expenditures including the expenses of PPRs. This issue can be analyzed in equation 21 as well:

\[ \frac{\partial G_p^*}{\partial \xi} > 0, \quad \frac{\partial^2 G_p^*}{\partial \xi^2} > 0 \]  

\[ (29) \]

Figure (3): the relationship between government budget (\( \xi \)) (surplus and deficit), and the government spending on PPRs (\( G_p^* \))

5. Calibration

5.1 The Optimal Government Spending on PPRs

In this section, taking 3 scenarios into consideration, we have analyzed the different amounts of growth rate (of the expenses) for PPRs based on the various amounts of economic growth rate. We replace the values of Table 3 in equation 17, and draw the figure of the growth rate of the government spending on PPRs on the basis of various amounts of economic growth rate (Figure 4). Considering the
Figure 4, we can observe that:
1. In order to have a higher growth rate, the growth rate of government spending on PPRs should also increase.
2. In order to have 6% growth rate, given the 2nd scenario, the growth rate of the PPRs should be equal to 11% at steady state.
3. At given economic growth rate, with a decrease in social intelligence of PPRs (a shift from scenario 1 to 3), the growth rate of government expenses on PPRs will increase.

Table (3): the amounts of parameters required for calculating the optimal amounts of the expenses related to PPRs

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social intelligence respect to property rights $(1-I)$</td>
<td>0.8</td>
<td>0.5</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Long run economic growth $(g^*)$</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Inverse of intertemporal elasticity of substitution $(\theta)$</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Discount rate $(\rho)$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Firm’s output elasticity respect to capital $(\sigma_k)$</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Firm’s output elasticity respect to government services $(\sigma_G)$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
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<tr>
<td>Depreciation rate $(\delta)$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Share of government deficit in GDP $(\xi)$</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.03</td>
<td></td>
</tr>
<tr>
<td>Income tax rate $(t)$</td>
<td>0.066</td>
<td>0.066</td>
<td>0.066</td>
<td></td>
</tr>
</tbody>
</table>
5.2 The Optimal share of Spending on PPRs in total spending

In previous part, given that the growth rate 6% (and with a consideration of other variables), the growth rate of PPRs was calculated. In this section we calculate, the amount of the spending on PPRs. By replacing the values of Table 3 in equation 21, we have drawn the optimal amount of spending on PPRs in Figure 5, based on GDP and given the 3 scenarios. We can observe that with an increase in GDP, the amount of the spending on PPRs will also increase. For instance, With 2nd scenario and given that the amount of GDP equal to 2.89 thousand trillion Rials (Nominal GDP in 2008) the optimal amount of government spending on PPRs will be equal to 0.26 thousand trillion Rials. Also based on the figure we can deduce that GDP is less than threshold level.
5.3 Calculating the share of government spending on PPRs in total government expenditures

In the two last parts, we did evaluate the government spending on PPRs and its growth rate. In this section we calculate the optimal amount of the share of the government spending on PPRs in the total government expenditure. Based on the equation 25 and 22 and the calculations carried out in section 4.4, we’ll have the equation 30. This equation indicates the share of government spending on PPRs in total expenditure, at the steady state.

$$\gamma^* = \frac{\left[ (1 + \xi - t) \right]^{-\theta} \sigma_C \left( 1 - \frac{\sigma_k \gamma^* + \sigma_k \delta}{\theta \gamma^* + \delta + \rho} \right)^{-\theta} \eta}{Y^{1+\frac{1}{\delta}}}$$

Given that the amounts of parameters from scenario 2, in Table 3, are inserted in equation 30, we can, draw the share of government spending on PPRs in total government expenditure, at the optimal point. As it is inferred from Figure 6, based on the GDP in year 2008, 31% of government expenditures should be spent in PPRs. This is in spite of the fact that this share is, on average, around 5.8%.
6. Concluding Remark

In this paper, a concept called “social intelligence with respect to PPRs” was initially introduced, and then the relationship between this variable and the level of growth rate of government spending on PPRs was extracted. Then the optimal amounts of the level and the growth rate of government spending on PPRs and its share in the total government expenditure were calculated. Finally, this model was calibrated on the basis of data in Iranian economy.

The results show that whenever the public have a high level of social intelligence about property rights, the government can reduce its expenditure, which in turn means a reduction in the growth rate of spending on PPRs. It means that the growth rate of government spending on PPRs is descending function of social intelligence about property rights.

We find whenever production is less than threshold level, with an increase in social intelligence, the spending on PPRs will also increase. If social intelligence of property rights is low, the growth rate of government spending on PPRs will be increasing. Also in case of a high degree of social intelligence with respect to property rights, growth rate of government spending on PPRs will be decreasing.

We show that with an increase in budget deficit, the government
reduces the spending on PPRs, and with an increase in budget surplus it increases that expenditure. This results are compatible with pro-cyclically behavior of fiscal policy in developing countries and Iran\textsuperscript{24}.

In case of an increase in budget deficit, the government reduces the spending on PPRs. This result is also compatible with the pro-cyclical behavior of fiscal policies in developing countries.

More ever, the results of calibrating the model indicated that to have an economic growth rate at 6%, the amount of government spending on PPRs should be 0.26 thousand trillion Rials, which contains 31% of the total government spending at the steady state.

In order to make the results more realistic, the following suggestions are offered for further research:

In this model it was assumed that only government is responsible for PPRs. It could be assumed that both public and private sectors pay the expenses for PPRs. In other words, the model can be expanded in a way that, a combination of legal centralism view and private ordering view is considered. By doing this one can calculate the optimal public and private shares of the expenditure on PPRs.

Given the assumptions of this model, one can drive the saddle path, and assess the policies required for transition of economy towards this path in order to reach a steady state. Finally, in addition to the equations of motion considered in this study, we can consider an equation of motion for social capital and solve the model.

Endnotes

1. It is assumed that legal centralism view is followed. Based on this view it is assumed that, property rights are defined and devured by the government. In contrast to this view, in private-ordering view the economic agents, with the aim of securing mutual interests in exchanges, do themselves invest, regardless of government’s role in PPRs.

2. In the studies of Turnovsky (2000), Chang (1999), Baier and Glomm (2001) and Agénor (2008) the government services introduced in utility functions as additive separable formats.

3. In order to work out this equation Rennani et al. (2008) have made use of the fiscal rule of budget deficit for government. Also they have considered two different function for public and private
sectors’ production. Given the dominant situation in the economy of developing countries, these researchers have also inserted the x-efficiency in public sector’s production equation, and legal system efficiency in private sector’s production function. For more information refer to Rennani et al. (2008, pp. 185-193)

4. Based on the definition of derivative and production function we have:

\[
\frac{\partial Y}{\partial K} = \frac{\sigma_Y Y}{K}, \quad \frac{\partial Y}{\partial H} = \frac{(1-\sigma_H)Y}{H}, \quad \frac{\partial Y}{\partial G_p} = \frac{\sigma_G Y}{G_p}
\]

5. Given the steadiness of parameters in the left-hand of equation (14), we will have:

I. \( g_y = g_K \)

We divide the equation (13) by \( K \), and with a little simplification the equation 2 is achieved:

II. \( g_c = g_K \)

Hence, considering the equation 2 and 3 in the long term balance of consumption growth rates of, accumulation of physical capital and production will be equate.

III. \( g_y = g_K = g_c = g^* \)

6. We have:

\[
(1-\lambda) \left( \frac{\dot{g}_K}{\lambda} + g_y \right) \overset{(10)}{=} (1-\lambda) \left( \frac{\dot{g}_K}{\lambda} - \rho = -(f \frac{\sigma_y Y}{K} - \delta) + g_y \right) \overset{(14)}{=} \]

\[
(1-\lambda) \dot{g}_K - \rho = -(f \sigma_k \frac{\theta g_c + \delta + \rho}{f \sigma_k} - \delta) + g_y
\]

\[
\Rightarrow (1-\lambda) \dot{g}_K = -\theta g^* + g^* \Rightarrow g_{\dot{g}_c} = \frac{1-\theta}{1-\lambda} g^*
\]

7. This equation is calculated by the following mathematical process:
\[\dot{K} = fY - \delta K - C \quad \Rightarrow \quad \dot{K} = \frac{fY - \delta K}{K} \]

\[g_K = \theta g_e + \delta + \rho - \frac{\delta - C}{K} \Rightarrow \frac{C}{K} = \frac{\theta g_e + \delta + \rho - \delta - g_K}{\sigma_k} \]

\[C = \frac{(\theta - \sigma_k)g^* + (1-\sigma_k)\delta + \rho}{\sigma_k} \Rightarrow C = \frac{(\theta - \sigma_k)g^* + (1-\sigma_k)\delta + \rho}{\sigma_k} K \quad (a)\]

\[f = \frac{\dot{g}_K + \delta + \rho}{\sigma_k} \Rightarrow \frac{f}{\sigma_k} = \frac{\theta g^* + \delta + \rho}{\sigma_k} K \quad (b)\]

\[C = fY(1-\sigma_k g^* + \sigma_k \delta) \quad \text{8. Given the following mathematical calculation we have:}\]

\[G^*_p = \left[\frac{Y^{\sigma - 1}}{\eta} \left(\frac{1 - \sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho}\right)^{\frac{1}{\eta}} \right]^{1/\eta} \Rightarrow \]

\[\ln G^*_p = \frac{1}{1-l} \ln \left[\frac{Y^{\sigma - 1}}{\eta} \left(\frac{1 - \sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho}\right)^{\frac{1}{\eta}} \right] \Rightarrow \]

\[\frac{\partial G^*_p}{\partial (1-l)} = -\frac{G^*_p}{(1-l)^2} \ln \left[\frac{Y^{\sigma - 1}}{\eta} \left(\frac{1 - \sigma_k g^* + \sigma_k \delta}{\theta g^* + \delta + \rho}\right)^{\frac{1}{\eta}} \right] \]

\[9. \text{That is because there has been enough accumulation necessary for PPRs.}\]

\[10. \text{By taking derivative of the first derivative } \left(\frac{\partial G^*_p}{\partial (1-l)}\right), \text{we will have:}\]
\[ \frac{\partial G^*_p}{\partial (1-\ell)} = - \frac{G^*_p}{(1-\ell)^2} \ln [Y^{\alpha-\beta} \left(1 + \xi - \ell \right)^{1-\beta} \sigma_{\ell} (1-\sigma_k g^* + \sigma_k \delta)^{\beta} ] \Rightarrow \]

\[ \frac{\partial^2 G^*_p}{\partial (1-\ell)^2} = - \frac{G^*_p}{(1-\ell)^3} \ln [Y^{\alpha-\beta} \left(1 + \xi - \ell \right)^{1-\beta} \sigma_{\ell} (1-\sigma_k g^* + \sigma_k \delta)^{\beta} ] + \]

\[ 2 \frac{G^*_p}{(1-\ell)^2} \ln [Y^{\alpha-\beta} \left(1 + \xi - \ell \right)^{1-\beta} \sigma_{\ell} (1-\sigma_k g^* + \sigma_k \delta)^{\beta} ] = 0 \Rightarrow \]

\[ \ln [Y^{\alpha-\beta} \left(1 + \xi - \ell \right)^{1-\beta} \sigma_{\ell} (1-\sigma_k g^* + \sigma_k \delta)^{\beta} ] = -2 \Rightarrow \]

\[ 1 - l = \ln [Y^{\alpha-\beta} \left(1 + \xi - \ell \right)^{1-\beta} \sigma_{\ell} (1-\sigma_k g^* + \sigma_k \delta)^{\beta} ]^{\frac{1}{1-\beta}} \]

11. Equation 26 is calculated in this way:

\[ \gamma^* = \frac{\beta}{(t-\xi)} Y^{\frac{1-\beta}{t-\xi}} \Rightarrow \ln \gamma^* = \ln \frac{\beta}{(t-\xi)} + \frac{1-\theta}{1-l} \ln Y^* \Rightarrow \frac{\partial \gamma^*}{\gamma^*} = \frac{(1-\theta)(1-l)}{(1-l)^2} \ln Y^* \]

\[ \frac{\partial Y^{*}}{\partial (1-l)} = - \frac{(1-\theta)Y^*}{(1-l)^2} \ln Y^* \]

12. All numerical calculations in the fourth and fifth sections have been carried out using MATLAB 10.1 and Excell.

13. The average annual growth rate in the fourth development plan is considered to be 8%. However, given the author’s studies, the 6% growth rate appears to be more realistic. In any case the figure (1) is drawn for both 6% and 8% growth rate.

14. In most articles, the risk aversion coefficient (inverse of the elasticity of substitution) in agricultural sector had been calculated. This amount is average of the calculations in various studies. In Abdoli (2009) this rate is measured to be 0.07

15. The figures are graphed on the basis of different depreciation rates (e.g. 0.06 and 0.05). The results show that these rates do not have a significant effect on figures and optimal amounts.

16. \[ z_{2007} = \frac{T_{2007} - G_{2007}}{Y_{2007}} \]

17. \[ t_{2007} = \frac{(government\ income\ from\ tax)_{2007}}{(GDP)_{2007}} \]

18. In underdeveloped countries, the level of social intelligence with
respect to property rights is very low (near zero) and in developed countries it is high (near one). Since Iran is a developing country, it is assumed that the level of social intelligence with respect to property rights is equal 0.5. Based on figure 2 this seems to be a logical level.

19. We could also investigate the relationship between consumer preferences and government’s budget deficit. In summary we can analyze the government expenses in PPRs as a function of social intelligence and government’s budget surplus or deficit. On the other hand it’s been indicated that discount rate is a decreasing function of government spending on Property rights in the case of an increase in government’s budget deficit, the amount of the expenses on PPRs will be decrease. With a decreasing in expenses, discount rate will increase.

20. This section is an application of the calculations in section 3
21. The data source are noted in table 1 and 2
22. The graph for all scenario have overlap. We have avoided drawing all of them here.
23. Rennani, et al. (2008, PP. 199)
24. This result also confirmed in Samadi and Oujimehr (2012)

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