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Modeling and Experimental Testing of Asymmetric Information Problems in Lease and Hire Contracts: A Study Based on Contract Theory

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Abstract

The present study is to examine lease and hire contracts in the Islamic setting of Iran. The study is further to analyze the asymmetric information problems in relation to these contracts. To this end, the features of lease and hire contracts, experimental characteristics revealed in some previous studies in Iran were examined through the use of library method. Different aspects of asymmetric information, namely hidden information and hidden action, were then mathematically modeled via contract theory. The resultant model indicated a list of optimal lease and hire contracts in transactions which can solve asymmetric information problems, such as adverse selection and moral hazard, through removing participation and incentive compatibility constraints. Finally, the optimal contract was determined with hypothetical parameters in the experimental analyses and through the use of LINGO software. Based on the findings, the main models provided for every transaction were solvable and the optimal contracts were obtainable. Experimental contracts show that the lessor has to set security deposits of tenant type θ_L lower than tenant type θ_H and set higher monthly rent for tenant type θ_L in lease contracts, and principal has to set wage of agent type θ_L lower than agent type θ_H and set higher length of contract for agent type θ_L in hiring contracts to solve asymmetric information problems.

1. Introduction

In the mainstream of Islamic-Iranian economic sciences, economic transactions, especially buying, selling, renting and cooperating have been analyzed in the frameworks of markets and supply and demand systems. Due to self-regulation of the market, however, there has been no possibility to consider native parameters or Islamic cognition in these analyses. It just receives data and creates outcomes like a black box. Even using open source analyses, such as

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econometrics and supply-demand estimations, the scholars have not considered native and Islamic parameters. However, the only way for scholars to include their ideas in the relevant models seems to be through consideration of these sorts of data. It means that scholars need to play their roles with the fixed “rules of the game”.

Stiglitz (2006) has claimed that the existence of incomplete information, because of distortion of information and incentive, as the pillars of economic transactions, can threat the efficiency and dynamicity of market system, in which the Arrow-Debreu assumptions are somewhat unrealistic. Accordingly, “game theory, mechanism design and contract theory have been introduced into economics to increase of scholars' capability for defining and changing 'rules of the game' and to make them expert in considering native or ideological parameters in the analysis process (Borgers et al., 2015). In the context that is based on bilateral and contractual transactions, mathematical models are completely useful to describe different dimensions of a transaction, because of flexibility of objective function (as the goal of participation in transaction) and its form, parameters and constraints.

Lease and hire contracts are among the most common agreements in Iran which have special characteristics affected by Iranian native or Islamic culture. Analyzing these transactions and facing the existent challenges and problems, e.g. asymmetric information problem, and conducting them, can be considered as an important step in enhancing efficiency of transactions .

Contract theory is the most popular tool for analyzing bilateral transactions like leasing or hiring. Therefore, the main objective of the present study is to analyze lease and hire transactions through the use of contract theory and to help solving such asymmetric information problems as adverse selection and moral hazard. To this end, the theoretical foundations of contract theory and leasing or hiring transactions will be presented in the second section of this study. The third section is an attempt to provide the readers with the modeling of these two types of transactions through defining their special characteristics in Iran, to present a mathematical model for each, and to determine the form of optimal contracts which make the incentives of both parties compatible and consistent. In the fourth section focuses on the solution to the main problem and determination of the sample optimal contract, through using hypothetical parameters and mathematical programming model in the software, to show how solvable and obtainable to optimal contracts are. The final section of the present study will present the readers with some conclusions regarding the resultant model.

2. Theoretical foundations

The present study benefits from contract theory as the theoretical framework for modeling different aspects of asymmetric information. Contract theory contains three major parts: a) incentive theory, b) transaction costs theory and c) incomplete contracts. Incentive theory analyzes the incentives problems, which are caused by asymmetric information, solves adverse selection and moral hazard problems, reveals the hidden information, and prohibits the hidden action through designing incentive compatible contracts (Brousseau & Glachant, 2002). Contract theory also has a close relationship with mechanism design and game theory, which besides the transactional approach, can be used for creating and managing incentives (Bolton & Dewatripont, 2005).

Adverse selection was first theoretically considered in models of asymmetric information by Akerlof (1970) who conducted a survey on “the market for lemons” and explored information problems in second-hand car markets, where sellers, as compared to buyers, have a better knowledge of whether their car has high quality or not (i.e. be a lemon). Akerlof paper officially started a new stream known as contract theory, and then models were expanded by Oliver Hart and Bengt Holmström who won the Noble prize in 2016.

One of the outstanding studies on modeling lease contract has been done by Benjamin et al. (1998) who examined the pricing of rental contracts for two types of renter households. Using empirical test and econometrics, they studied the situation in which landlord rents his house. The results showed that similar risky loans, renting with little or no security deposit makes more returns. Although the researchers have benefitted from an efficient research methodology, the model suggested by them is incompatible with Iranian lease market. They further have not examined all aspects of asymmetric information, which are discussed in the present study.

The basic model of contract theory, used in this study, and the form of optimization problems and constraints has been adapted from Laffont and Martimort (2001) and Bolton and Dewatripont (2005) which are amongst the most outstanding works in the realm of contract theory. The idea of modeling hire contracts based on contract theory has come from Brousseau and Glachant (2002) and Salanie (2005) who have intuitively described asymmetric information forms in the most popular economic contracts.

Lease and hire market characteristics which are used in the mathematical models, provided in the sections, and are considered as presumptions are based on library method and real contracts drawn up in Iran. The hypothetical parameters, which are used in the present study's experimental analyses, have even been tried to be compatible with the Islamic-Iranian market. They are not, however, based on any field study. Using these theoretical foundations, we will start to model lease and hire contracts in the next section.

3. Modelling

In this section, the characteristics of lease and hire contracts in Iran will be introduced first. The present research will, then, define the problem and mathematically model these contracts to find ways in resolving the asymmetric information problems.

3.1 Lease Contract Modeling

Lease contract is a transaction between lessor, as principal, and tenant, as agent where the subjects of lease are objective things such as house, car, and furniture. Before modeling, the following research had to attend to several points. First, because of various details of leasing in any contract, they had to analyze the model through conducting a case study, on house leasing, for example, so that they would, then, be able to generalize the model to other leasing contracts. Second, they had to initially study the real characteristics of these transactions in the society, of Iran, and find the required parameters and variables. Third, in the last step and before modeling, they had to simplify the model and make some assumptions to start from and gradually release them to realize the model.

In Islamic-Iranian lease contracts, the tenant needs to provide the lessor some information about his/her family and other characteristics. The Lessor, then, offers a combination of monthly rent and security deposit as mortgage[1] which might be either accepted or refused by the tenant.

The market characteristics

- The rent is paid monthly and the contract is usually terminated after one year and cannot be changed or renegotiate in the period of contract.
- Security deposit is usually paid at the beginning of the period of transaction and repaid to the tenant when the contract expires.
- The lessor offers the contract and the tenant accepts it or refuses it, i.e. the tenant cannot interfere in writing the contents of a contract.
- Tenant income has to be more than the monthly rent and his savings has to be more than the security deposit.
- Tenants are not homogenous and have different characteristics.
- The tenant characteristics affect the amount of monthly rent and security deposits. Therefore, different tenants prefer different contracts.

The parameters of the model

- "n" refers to the payment frequency rate, which is usually 12.
- "r" refers to the monthly rent amount in the contract.
- "d" stands for the amount of the security deposits in the contract.
- "y" is the income of the tenant.
- "p" stands for the probability of any damage to or unusual depreciation of depreciating the house ($p_H > p_L$).

[1] Security deposit taken because its possible tenant make damage or unusually depreciate the house. Of course there is a transaction between security deposit and monthly rent, because of the interest rates, and this transaction rates depend on interest rate, the characteristics of the house, and the position of the house in the city.

- "φ^V" is the level of verifiable damage, which can be included in the contract ($\varphi_H^V > \varphi_L^V$).
- "φ^N" is the level of non-verifiable damage, which cannot be included in the contract ($\varphi_H^N > \varphi_L^N$).
- "θ_H" refers to the type of tenant who causes a high level of, verifiable and/or non-verifiable damage to the house or unusually depreciates the house.
- "θ_L" stands for the type of tenant who makes a low level of damage to the house or unusually depreciates the house.
- "δ" is the rate of time preference of money and $0 < \delta < 1$ (which might be different for different people).
- "$u(\cdot)$" is the utility function of money for the tenant.
- "$U(\cdot)$" refers to the utility function of money for the lessor.

Initial simplifying assumptions

- First, it was assumed that there is complete information and the lessor knows types of tenants.
- The lessor plays the role of principal and the tenant plays the role of agent in the contract.
- There are only two types of agents (tenant) in the market (θ_H , θ_L).
- There is no financial constraint on the tenant, regarding the amount of rent or security deposit, and every optimal contract can be accepted by the tenant.
- There is a perfect substitution between the rent and the security deposit.
- It was also assumed that the amount of security deposit is more than the verifiable damage ($d \geq \varphi^V$).
- The lease is a one-shot transaction and the information within a contract cannot be used in / another contract and the previous costs cannot be settled in the next contract.
- The lease market is a perfectly competitive market.
- Asymmetric information is only considered as the probability of making damage to the house by the tenant.

3.1.1 Step one: Optimal contract under complete information

If the tenants pay the rent r every month and the security deposit d at the time of entering into a contract, the utility function of tenant will be:

$$u(\theta_i) = u\big(c(H)\big) + u(-d) + u(y-r) + \delta u(y-r) + \cdots + \delta^{n-1}u(y-r) + \delta^n(1-p_i)u(d) + \delta^n p_i u\big(d - \varphi_i^V\big) \tag{1}$$

As you can see, the tenant uses the benefits of house and earn $c(H)$. Furthermore, s/he pays a security deposit and bears the disutility $u(-d)$. The rents which is subtracted from the income, reduces his utility to $u(y-r)$. When the contract terminates, the tenant makes no verifiable damage to the house with a probability level of $(1-p_i)$ and then gets all of the security deposit back which makes utility equal to $\delta^n(1-p_i)u(d)$[1] for him.

Based on the structure of tenant's utility function in the $r-d$ Cartesian coordinate system, indifference curves are concave and increase utility when they are closing to the origin. Because lower levels of rent and security deposit make more utility for tenants and because an extreme combination, e. g. high rents without security deposit or vice versa, is not preferred, then utility function concave[2].

Tenant problem maximizes his/her own utility function and the only constraint on him/her is the financial constraint, i.e. his/her monthly rent has to be more than his/her monthly income and the security deposit has to be more than his/her savings). Consequently, we have:

$$\max_{r,d} \quad u\big(c(H)\big) + u(-d) + u(y-r) + \delta u(y-r) + \cdots + \delta^{n-1}u(y-r) + \delta^n(1-p_i)u(d) + \delta^n p_i u\big(d-\varphi_i^V\big)$$

$$s.t \qquad y-r>0$$
$$S-d\geq 0 \tag{2}$$

One can find out the marginal rate of substitution between r and d for a tenant through solving this problem.

The Lessor' (principal') s utility function is:

$$U(P) = U[(r-\delta C) + (\delta r - \delta^2 C) + \cdots + (\delta^{n-1}r - \delta^n C) + G(\Delta P_H)] + U(d) - \delta^n(1-p_i)U(d+\varphi_i^N) - \delta^n p_i U((d-\varphi_i^V) + \varphi_i^N + \varphi_i^V) \tag{3}$$

where the lessor earns positive utility from the rent r and disutility from opportunity cost of capital of buying the house C and also positive utility from a gain made by the change in the price of house $G(\Delta P_H)$. Also s/he earns positive utility from security deposit at the beginning of the period and disutility from paying back at the end of the period. The simplified utility function will be:

$$U(P) = U[\pi] + U(id) - \delta^n U\big(\varphi_i^N\big) \tag{4}$$

The first part of this utility function is related to the profits on leasing the house and the second part is related to having security deposits, e.g. interest rate. If the utility function of money for principal is unit function, then we will have:

$$U(P) = \pi + \delta^n id - \delta^n \varphi_i^N \tag{5}$$

where i refers to the interest rate of money. Under complete information, the principal problem is increasing or maximizing the profit on leasing the house subject to participation constraint of tenant in the contract. Then

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[1] Because non-verifiable damage would subtract from the security deposit then wouldn't affect the utility of tenants directly and just from consumption function of benefit of the house $u\big(c(H)\big)$ it would be included.

[2] According to real statistics of the lease market in Iran, combination between security deposit and rent are more demanded than high rents without security deposit or high security deposit without any rent. Then we can conclude that middle combinations of rent and security deposit are preferring to extreme combinations.

$$\max_{r,d} \quad (r - \delta C) + (\delta r - \delta^2 C) + \cdots + (\delta^{n-1} r - \delta^n C) + G(\Delta P_H) + \delta^n id - \delta^n \varphi_i^N$$

$$s.t \qquad u(\theta_i) \geq 0 \tag{6}$$

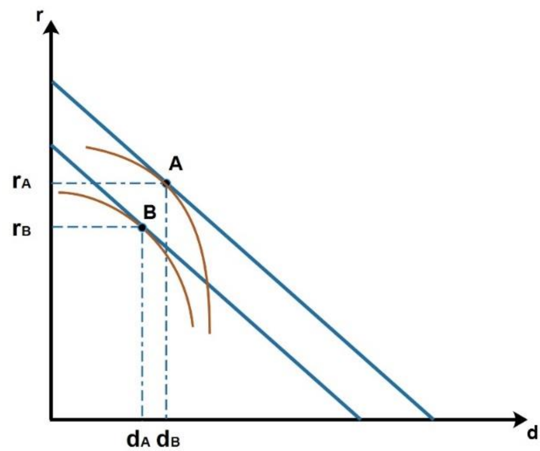
where you can find the marginal rate of substitution between r and d for the lessor (principal).

The important point here is that if the market was monopoly, i.e. if there was only one lessor in the market, then the tenants have to pay all their income for the rent and all their savings for the security deposit in the optimal condition. In such a condition, you have to search about justice of leasing through considering market structure but not the contract.

In a perfectly competitive leasing market, the long-run profit is zero and the relation between rent and security deposit is

$$r = (1-\delta)\left[\frac{C\delta}{1-\delta} + \Delta p_H + \delta^n \varphi_i^N\right] - i\delta(1-\delta)d \tag{7}$$

Graph 1 shows the optimal contracts for each type of tenants. The lines are represented by Equation 7 and the concave curves[1] by Equation 6.



Graph 1.Optimal lease contracts under complete information

In Graph 1, the decreasing lines are the locus combination of r and d in such a way that the lessor' (principal') s profit in a perfectly competitive market is zero. The slope of contract for every tenant is $i\delta(1-\delta)$ and the intercept is $(1-\delta)[C\delta + \Delta p_H + \delta^n \varphi_i^N]$, and because $\varphi_L^N < \varphi_H^N$, then it can be recognized that the zero profit line for the tenant who has caused a low level of damage to the house (θ_L) is lower than the zero profit line for the tenant who has caused a high level of damage (θ_H). Additionally, we know that the indifference curve is

[1] Because the probability of damage for θ_H is more than θ_L, then extreme combination will reduce utility of θ_H more than θ_L. Then it is logical that the concavity of indifference curve for θ_H become more than θ_L.

steeper for θ_H. Point A and B represent the optimal contracts for θ_H and θ_L , respectively and are (r_A, d_A) and (r_B, d_B).

3.1.2 Step two: optimal contract under asymmetric information

The first unrealistic assumption in our model was complete information. In reality, however, the lessor does not know about the tenant's type. Of course, there is a proxy, such as the number of children and their age, for guessing the type; however, the tenant behavior regarding making damage to the house and the probability of it are related to his/her incentives to leasing the house. These incentives are indeed the tenant's private information. Hidden information can cause adverse selection problem for the lessor so that he cannot choose the optimal contract easily.

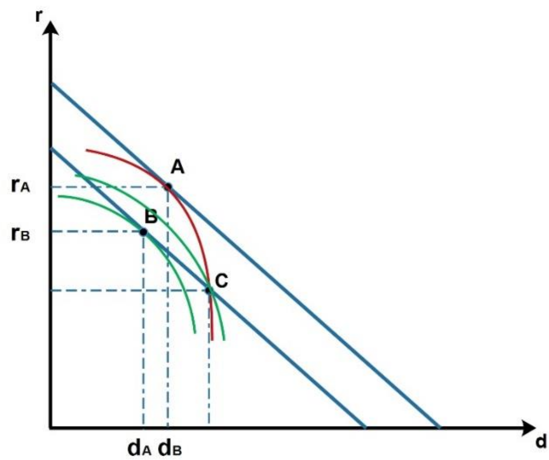
As one can see in Graph 1, the lessor determines a higher rent and security deposit for tenant θ_H. Then, under asymmetric information, tenant θ_H introduces himself as θ_L so that he can pay a lower rent and security deposit. To solve this problem and to make the tenants reveal themselves honestly, the principal has to remove incentive compatibility constraints in the contract in such a way that every tenant can earn a higher utility through choosing his own contract. The constraints are:

$$u(\theta_{HH}) \geq u(\theta_{HL})$$
$$u(\theta_{LL}) \geq u(\theta_{LH}) \tag{8}$$

where $u(\theta_{HL})$ represent the amount of tenant θ_H's utility when she/he chooses the contract of θ_L and $u(\theta_{HH})$ refers to the amount of tenant θ_H' utility when she/he chooses his own contract. The incentive compatibility constraints make the lessor sure that every tenant earns more utility through choosing his own contract, and thus she/he reveals themselves honestly. The lessor's problem, then, will be:

$$\max_{r,d}(r-\delta C)+(\delta r-\delta^2 C)+\cdots+(\delta^{n-1}r-\delta^n C)+G(\Delta P_H)+\delta^n id-\delta^n\varphi_i^N$$
$$s.t \quad u(\theta_i)\geq 0$$
$$u(\theta_{ii})\geq u(\theta_{ij}) \quad ; \quad i,j=H,L \tag{9}$$

This is shown Graph 2.



Graph 2.Optimal contracts under asymmetric information

In this situation, point B is not optimal and the lessor has to offer the contract C to tenant $\theta_L$to remove the incentive compatibility constraints. Then because tenant θ_H is indifferent between A and C, she/he has no incentive to hide his type and he will introduce himself honestly[1]. Disability of the lessor to recognize the type of tenants can reduce the utility of tenant (θ_L) who causes a low level of damage to the house Indeed, asymmetric information gives no information rent to tenant θ_L and only imposes costs on tenant θ_L. Even in this situation, tenant θ_L prefers contract C over contract A.The lessor, then, solves the asymmetric information problem and finds the hidden information through offering contracts A and C.

3.1.3 Step three: adding assumption $\mathbf{d < \varphi^V}$

It was assumed in the base model that the security deposit is more than verifiable damage. But in reality, it is not always true. In this case, the utility function of lessor will be determined through Equation 10:

$$U(P) = U[(r-\delta C) + (\delta r - \delta^2 C) + \cdots + (\delta^{n-1} r - \delta^n C) + G(\Delta P_H)] + U(d) - \delta^n (1-p_i) U(d+\varphi_i^N) - \delta^n p_i U(\varphi_i^V - d) - \delta^n p_i U(d+\varphi_i^N) \tag{10}$$

If the tenant does not make verifiable damage (φ_i^V) to the house with a probability level of $(1-p_i)$, then the lessor pays all the security deposit back to the tenant. But if the tenant makes damage (φ_i^V) to the house with a probability level of (p_i) because $d < \varphi^V$, then, the lessor has to spend all the security deposit to compensate the part of damage (φ^V) which has been made to the house and pay (φ_i^N) which makes disutility equal to $\delta^n p_i U(d+\varphi_i^N)$ and the

[1] The indifference curve that pass from C and A points is related to tenant θ_H and the other indifference curve that just pass point C is related to tenant θ_L.

rest of (φ^V), i.e. $(\varphi_i^V - d)$, which makes disutility equal to $\delta^n p_i U(\varphi_i^V - d)$. Simplifying Equation 10, we will have:

$$U(P) = U[\pi] + U(id) - \delta^n U(\varphi_i^N) - \delta^n p_i U(\varphi_i^V - d) \tag{11}$$

Similar to Equation 5, with the assumption lessor's unit utility function, we will have:

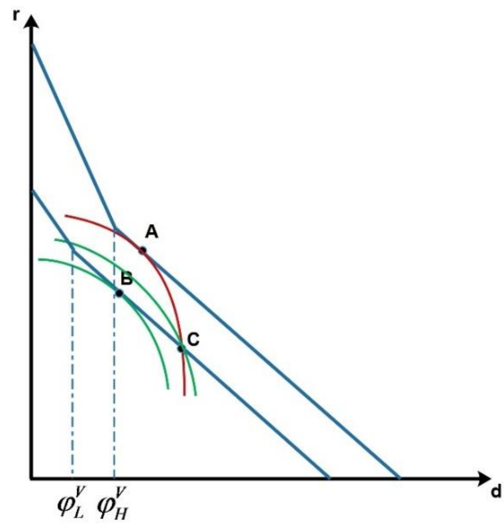
$$U(P) = \pi + \delta^n id - \delta^n \varphi_i^N - \delta^n p_i(\varphi_i^V - d) \tag{12}$$

Then there is a new relationship between r and d which leads to zero profit of lessor in the perfectly competitive market, i.e.

$$r = (1-\delta)\left[\frac{C\delta}{1-\delta} + \Delta p_H + \delta^n \varphi_i^N\right] - (1-\delta)(i\delta + p_i\delta^n)d \tag{13}$$

Comparing Equation 13 with Equation 7, we can observe that the slopes of zero profit lines are more than before, now. In addition, because $p_L < p_H$, the zero line profits for tenant θ_H is steeper than those for tenant θ_L.

It is worth mentioning here that Equation 7 is related to the part of graph with $d \geq \varphi^V$, and Equation 13 is related to the part of graph with $d < \varphi^V$. Then, in general, as it has been shown in Graph 3, zero profit lines become kinked:



Graph 3.Optimal contracts under asymmetric information with $d < \varphi^V$

If the points of optimal contracts are on the right side of φ^V, the contractual parameters (r و d) will be adapted from Equation 9 and if the points of optimal contracts are on the left side of φ^V, the contractual prompters will be adapted from Equation 14:

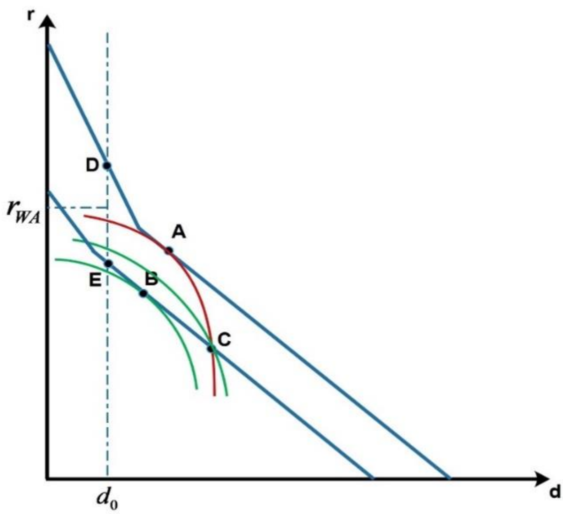
$$\max_{r,d} \; (r - \delta C) + (\delta r - \delta^2 C) + \cdots + (\delta^{n-1} r - \delta^n C) + G(\Delta P_H) + \delta^n id - \delta^n \varphi_i^N - \delta^n p_i(\varphi_i^V - d)$$

$$\begin{aligned} s.t \quad & u(\theta_i) \geq 0 \\ & u(\theta_{ii}) \geq u(\theta_{ij}) \;\; ; \quad i, j = H, L \end{aligned} \tag{14}$$

The graph for Equation 14 is similar to that for Equation 3 with the only difference that the indifference curves are tangents to the zero profit lines in the steeper part.

3.1.4 Step four: adding financial constraints $\mathbf{d \leq d_0}$

It was assumed in the base model that there are no financial constraints on tenants and every optimal contract can be accepted by them. But in reality, it is not true and there may be some tenants with no sufficient income to pay the monthly rent and some others with no sufficient savings to pay all the security deposit, for example when the tenant cannot pay more than d_0. Then, there will financial constraints, like $d \leq d_0$, in the model. If the amount of d or r, as offered in the optimal contract, is less than the constraint, then there will be no change in the model. But if d or r is more than that, the constraint will be binding and has to be considered. This is shown in Graph 4:



Graph 4.Optimal contracts under asymmetric information and with financial constraints

As it can be observed in Graph 4, points D and E represent optimal contracts. But the most important problem here is that these two contracts do not remove incentive compatibility constraints. For instance, tenant θ_H is not indifferent between these two contracts. Then, the asymmetric information problem cannot be solved through offering the present list of contracts.

To solve this problem, we have to offer the same contract, such as (r_{WA}, d_0), to both tenant. r_{WA} refers to the weighted average of the two rents. The weight is the share of any type of tenant in the market. By doing this, lessor's profit becomes positive in some contracts, negative in some others, and zero in the long run.

3.1.5 Step five: relaxing assumption "perfect competition"

It was assumed that the leasing market is perfectly competitive but in reality, it is not the case. This is because houses in the market are not homogenous, in size, location, type, floor, and facilities, and tenants do not know about all the available opportunities for leasing their optimal house. This market is not, therefore, perfectly competitive and has nearly a monopolistically competitive structure.

Regarding this structure, in the short run, the moderated positive profits of the lessor can be replaced with zero in the model. Furthermore, there will be no other basic change in the model and only intercepts in the zero profit lines will turn into to specific positive amounts in the profit lines.

3.2 Hire Contract Modeling

In a hire contract, the principal is the person who hires someone to work for, entrepreneur, and the agent is the worker who is paid to do something.

The market characteristics

- One principal wants to hire one worker (agent) to do a specific rate of work.
- Workers are not homogenous in the market.
- A hire contract is a one-shot transaction and the information within one contract cannot be used in another contract and the previous costs cannot be settle in the next contract.
- The principal offers the contract and the agents can accept it or not.

The parameters of the model

- "q" is the production level offered by principal, which is the function of number of labor (L) and their skill and productivity levels (θ_i), which can be represented by $q_i = Q(L, \theta_i)$.
- "L" is the number of working hours.
- "θ_i" stands for the production cost which can reflect worker's level of skill.
- "W" refers to the level of wages and payments.
- "G" is a constant positive number which shows worker's utility of leisure.
- "l_i" stands for effective working hours.
- "$\bar{l}$" is the time that worker is potentially able to work.
- "β_L" is the share of low-skilled agents in the market and shows the probability of choosing the low-skilled by principal in the contract.
- "β_H" is the share of high-skilled agents in the market and shows the probability of choosing the high-skilled by principal in the contract.

<u>Initial simplifying assumptions</u>

- $Q(\cdot)$ represents concave function which is increasing in case of L but decreasing in case of θ_i as production cost.
- $\theta \in \{\theta_L, \theta_H\}$ where θ_H stands for the production cost of high-skilled agents and θ_L for the production cost for low-skilled agents where $\theta_L > \theta_H$.
- The hiring market structure is perfectly competitive.
- There is perfect substitution between leisure and work and $\bar{l} \geq l_i$ is assumed.
- $\beta_L + \beta_H = 1$.

<u>Asymmetric information forms</u>

- *Adverse selection as principal problem*: The situation in which the principal does not have enough information about the skill levels of agents.
- *Adverse selection as agent problem*: The situation occurs when macroeconomic variables, such as prices, change in the boom and recession of economy, and when only principals have precise information about market situation and know the exact amount of wages. In such a situation, agents have not enough information about these issues and face adverse selection problem.
- *Moral hazard as principal problem*: In most transactions, the principal cannot constantly monitor the agent and exactly analyze his/her working quality. The worker, then, can take hidden action, i.e. devote less effort to his work than what she/he has promised.

3.2.1 Step one: adverse selection as principal problem

In this case, the worker has precise information about his skill and productivity level. This information is of private type and hidden from the principal. Consequently, the principal has to determine the parameters of the contract in such a way that the agent will be able to provide him with his information. To this end, he has to solve principal problem in contract theory with participation and incentive compatibility constraints and find the optimal parameters, for instance optimal wage and production level.

The firm can choose a worker with a utility function of

$$u_i\left[W_i + G_i\left(\bar{l} - l_i\right) - \theta_i q_i\right] \tag{15}$$

Then, if the principal decides to maximize his expected utility, he has to solve this problem:

$$\max_{w_i, l_i} \quad \beta_H U[PQ(l_H, \theta_H) - w_H l_H] + \beta_L U[PQ(l_L, \theta_L) - w_L l_L] \tag{16}$$

where P is the price or value of a production for the principal. If the utility of nonparticipation in the contract, utility of leisure or other opportunities, is equal to $\bar{u}_i$ for the agent, then participation constraints, that make sure, agents will participate in the contract, will be:

$$u_L\left[W_L + G_L\left(\bar{l} - l_L\right) - \theta_L q_L\right] \geq \bar{u}_L \tag{17}$$

$$u_H\left[W_H + G_H\left(\bar{l} - l_H\right) - \theta_H q_H\right] \geq \bar{u}_H \tag{18}$$

But the most important problem here is that the principal does not know about skill and productivity levels of agents whom he hires and cannot determine the to be hired agents' optimal wage or optimal working hours for producing a certain amount of output. Then, she/he has to impose incentive compatibility constraints, on the agents, and make them to be involved in the problem. In this case, every worker will reveal his/her type honestly. Therefore, we have:

$$W_L + G_L\left(\bar{l} - l_L\right) - \theta_L q_L \geq W_H + G_L\left(\bar{l} - l_H\right) - \theta_L q_H \tag{19}$$

$$W_H + G_H\left(\bar{l} - l_H\right) - \theta_H q_H \geq W_L + G_H\left(\bar{l} - l_L\right) - \theta_H q_L \tag{20}$$

If the opportunity cost of the high-skilled agent is more than that of the low-skilled agent, then $G_H > G_L$. For a specific level of production to be achieved, the principal needs to hire the low-skilled agent rather than the high-skilled one, then $l_H < l_L$. In this case, incentive compatibility constraints will be enforceable and, at least, one of them will be binding. As a result, the principal problem will be:

$$\begin{aligned}
\max_{w_i, l_i} \quad & \beta_H U[PQ(l_H, \theta_H) - w_H l_H] + \beta_L U[PQ(l_L, \theta_L) - w_L l_L] \\
s.t{:} \quad & u_L\left[W_L + G_L\left(\bar{l} - l_L\right) - \theta_L q_L\right] \geq \bar{u}_L \\
& u_H\left[W_H + G_H\left(\bar{l} - l_H\right) - \theta_H q_H\right] \geq \bar{u}_H \\
& W_L + G_L\left(\bar{l} - l_L\right) - \theta_L q_L \geq W_H + G_L\left(\bar{l} - l_H\right) - \theta_L q_H \\
& W_H + G_H\left(\bar{l} - l_H\right) - \theta_H q_H \geq W_L + G_H\left(\bar{l} - l_L\right) - \theta_H q_L
\end{aligned}$$

Solving this problem, the principal can determine optimal contracts and parameters, such as optimal wage for every type of worker and optimal number of hiring hours, and thus he can obtain a list of contracts, such as $\{(W_H^*, l_H^*), (W_L^*, l_L^*)\}$ which satisfies both participation and incentive compatibility constraints.

3.2.2 Step two: adverse selection as agent problem

Changes in the economic situation and going through boom and rescission can leads to changes in returns of economic activities and also changes in wages and payments of the workers. Selling price for a product and product development costs change in different economic situations and employee productivity does too.

Considering the basic model parameters, production technology is assumed as $q_i = \theta_i Q(L)$, where θ_i is the stochastic value of production of workers in the boom (θ_H) or rescission (θ_L) and $\theta_H > \theta_L$. The probability of boom is equal to β_H and the probability of rescission is equal to β_L. As we know, according to microeconomic theory, the wage demanded by agents during the boom period is higher than that in the depression period, i.e. $l_H > l_L$. If the agents want to be hired by the employer, they have to remove employer's participation constraints during the boom or depression period. Accordingly, we have:

$$U[\theta_H Q(l_H) - W_H] > \bar{U} \tag{21}$$

$$U[\theta_L Q(l_L) - W_L] > \bar{U} \tag{22}$$

that $\bar{U}$ is employer's utility from not participating in the hire contract or opportunity cost of making the transaction. The problem here is that in any economic situation, selling price of products, product development costs, and some other macroeconomic variables are only observable for the employer (principal) and the worker (agents) does not usually have any precise information about these variables. Therefore, agents, workers, do not use their bargaining power and choose the wrong contract. Consequently, they face adverse selection problem and need to remove the employer's incentive compatibility constraints. Therefore,

$$\theta_H Q(l_H) - W_H \geq \theta_H Q(l_L) - W_L \tag{23}$$

$$\theta_L Q(l_L) - W_L \geq \theta_L Q(l_H) - W_H \tag{24}$$

When the employer tries to give a lower wage to the worker by claiming that they are going through depression, he needs to demand a lower level of effort from the worker, because demands usually decrease during a depression period. This synchronization decreases the return of employer's private information and the worker can use it for incentive compatibility. The agent (worker) problem, then, will be:

$$\max_{w_i, l_i} \quad \beta_H u\left[W_L + G_L\left(\bar{l} - l_L\right) - \theta_L q_L\right] + \beta_L U\left[W_H + G_H\left(\bar{l} - l_H\right) - \theta_H q_H\right]$$

$$\begin{aligned} s.t: \quad & U[\theta_H Q(l_H) - W_H] > \bar{U} \\ & U[\theta_L Q(l_L) - W_L] > \bar{U} \\ & \theta_H Q(l_H) - W_H \geq \theta_H Q(l_L) - W_L \\ & \theta_L Q(l_L) - W_L \geq \theta_L Q(l_H) - W_H \end{aligned}$$

3.2.3 Step three: Moral hazard as a principal problem

Bilateral hidden information problem can be solved if the hiring contracts are calculated following the first and the second steps. However, there is still the possibility for hidden action or moral hazard problem. If the employer does not effectively monitor the level and quality of the worker’s performance, the worker, then, does the lowest possible level of performance to maximize his/her utility. The major question here is can a contract be written in a way so that enough incentive for the worker's devotion of considerable effort to his/her job and consistent incentives would be guaranteed?

If the production level was observable and exactly reflected the worker's level of effort, then wages can simply be proportionate to production level. In such a situation, the worker has incentive to devote hard effort to maximize the production level and, as a result, his wage. But the point here is that, in most cases, devotion of more effort is not clearly apparent in the production level and can just increase the probability of a higher level of production[1].

Consider a risk neutral worker who is employed by one firm or employer. For simplicity's sake, consider only two production levels, namely high-level $\bar{Q}$ and low-level $\underline{Q}$). It is assumed that the level of effort belongs to collection $\{0, 1\}$, that the level of production is equal to $\overline{Q}$ with a probability of $\pi(e)$ and equal to $\underline{Q}$ with a probability $1 - \pi(e)$. Consequently, a new constraint needs to be entered into in the model so that the worker's incentives can be consistent with the employer's incentives.

If we assume that the employer can only reward a worker for his/her good performance and he cannot punish him because of his bad (poor) performance, then $w \geq 0$. The employer has to design an optimal program of wage, such as $\{(\underline{w}, \bar{w})\}$,to solve the optimization problem:

$$\max_{\{(\underline{w},\bar{w})\}} \quad \pi_1(\bar{Q} - \bar{w}) + (1 - \pi_1)(\underline{Q} - \underline{w}) \tag{25}$$

where π_1 refers to the probability of achieving a high level of production when the worker devotes a large amount of effort ($e = 1$). After solving the optimization problem, the employer can determine optimal wage(s). We know that optimal wages are appropriate to the achieved production level. Here, the employer also needs to remove the participation constraint to assure that the worker accepts the contract. As a result, the participation constraint will be:

$$\pi_1\bar{w} + (1 - \pi_1)\underline{w} - \psi(e) \geq \bar{u} \tag{26}$$

Because the wage rate is probabilistic, then the worker has an expected utility function. Regarding Equation 26, $\psi(e)$ refers to the amount of worker's utility from devoting effort. Obviously, then $\psi(0) = 0$.

When the relation between production level and effort level is probabilistic, depending wages on two levels of production, necessarily, cannot make the incentives of both parties consistent, because an agent's devotion of a low level of effort to his/her job may maximize his/her utility). The Employer has to make sure that the worker will choose to devote a high level of effort to his job ($e = 1$). Then, he has to enter another constraint into the problem:

$$\pi_1\bar{w} + (1 - \pi_1)\underline{w} - \psi(e) \geq \pi_0\bar{w} + (1 - \pi_0)\underline{w} \tag{27}$$

With regard to the employer's target function and the defined constraint, the general optimization problem to solve the moral hazard problem will be:

$$\begin{aligned} &\max_{\{(\underline{w},\bar{w})\}} && \pi_1(\overline{Q} - \overline{w}) + (1 - \pi_1)(\underline{Q} - \underline{w}) \\ &s.t.: && \bar{w} + (1 - \pi_1)\underline{w} - \psi(e) \geq \bar{u} \\ & && \pi_1\bar{w} + (1 - \pi_1)\underline{w} - \psi(e) \geq \pi_0\bar{w} + (1 - \pi_0)\underline{w} \\ & && w \geq 0 \end{aligned}$$

[1] Like laboratory and experimental tests (chemical, medical or engineering) that more effort not necessarily cause more exploration.

After solving this problem, the employer can determine the optimal wages which can solve the moral hazard problem.

4. Experimental Analyses

In the previous sections, contract relationships between principal and agent in different transactions were modeled. However, considering the fact that the model of concern in the present study is of a theoretical type, the present researchers had to make sure that the model is solvable. Therefore, the main models of this article were solved using LINGO software and hypothetical parameters[1], and the optimal contract was determined.

4.1 Lease Contract under Complete Information

Considerations for solving the model of the first step of section 3.1:

- Equation 6 is related to the main problem and its constraint utility function of tenant has been adapted from equation 2. Regarding optimization problem, the present researchers had to consider the expected utility function of lessor, obtained through multiplying the probability of realizing every types of tenant by the utility of leasing the house.
- The utility of renting the house for tenant θ_H is higher- than that for tenant θ_L.
- The probability level of making damage and the level of damage for tenant θ_H are higher than those for tenant θ_L .
- In programming the model, tenant's income was considered as zero to show that even if the utility of his monthly income has not been considered, his utility can be still positive due to his participation in the lease contract.
- The lease contract is made for one year which means 12 monthly rent payments ($n = 12$).

Taking all the above into consideration and after programming the model, under complete information, via the LINGO software, the following list of optimal contracts was obtained:

[1] With the field study these parameters can calculate from the real world in every country and replace with hypothetical prompters in this model.

Global optimal solution found:

Objective value:	171612.7
Infeasibilities:	0.000000
Total solver iterations:	0

Variable	Value	Reduced Cost
Q	0.7000000	0.000000
RH	824.7031	0.000000
SIGMA	0.9700000	0.000000
C	3333.333	0.000000
PH2	400000.0	0.000000
PH1	200000.0	0.000000
I	0.2000000	0.000000
DH	2000.000	0.000000
PHHN	2000.000	0.000000
RL	615.1279	0.000000
DL	1000.000	0.000000
PHLN	1000.000	0.000000
UCHH	10000.00	0.000000
Y	0.000000	0.000000
PH	0.7000000	0.000000
PHHV	2000.000	0.000000
UCHL	7000.000	0.000000
PL	0.6000000	0.000000
PHLV	1000.000	0.000000

where the calculated optimal contract is $(r_H = 824\ , d_H = 2000)$ for tenant type $\theta_H$and $(r_L = 615\ , d_L = 1000)$ for tenant type θ_L . These contracts well adapt to the theoretical foundations of the model and, as it has been shown in Graph 2, the amounts of rent and security deposit for the tenant type θ_H is larger than those for the tenant type θ_L.

4.2 Lease Contract under Asymmetric Information

The results of running equation 9 with above considerations in the LINGO software will be:

Global optimal solution found:

Objective value:	169743.5
Infeasibilities:	0.000000
Total solver iterations:	0

Variable	Value	Reduced Cost
Q	0.7000000	0.000000
RH	585.1279	0.000000
SIGMA	0.9700000	0.000000
C	3333.333	0.000000
PH2	400000.0	0.000000
PH1	200000.0	0.000000
I	0.2000000	0.000000
DH	2000.000	0.000000
PHHN	2000.000	0.000000
RL	615.1279	0.000000
DL	1000.000	0.000000
PHLN	1000.000	0.000000
UCHH	10000.00	0.000000
Y	0.000000	0.000000
PH	0.7000000	0.000000
PHHV	2000.000	0.000000
UCHL	7000.000	0.000000
PL	0.6000000	0.000000
PHLV	1000.000	0.000000

Therefore, the calculated optimal contract for tenant type θ_H is ($r_H = 585$, $d_H = 2000$) and it is for tenant type θ_L ($r_L = 615$, $d_L = 1000$). These contracts, too, well adapt to the theoretical foundations of the model and, as one can see in Graph 3, due to the information rent, the lessor has to set a smaller amount of security deposits and a higher level of monthly rent for tenant type θ_L, than tenant type θ_H, (using perfect substitution between security deposit and monthly rent). The results of the calculated model can verify this theory.

4.3 Hire Contract under Asymmetric Information

Considerations for solving the model of the second step of section 3.2:

- The production function is in Cobb–Douglas form, i.e. $Q(l_i, \theta_i) = l_i^{\alpha}\theta_i^{\alpha-1}$ where $0 < \alpha < 1$, and it is assumed to be increasing for l_i and decreasing for θ_i.
- The hypothetical parameters are considered as $P = 100$, $\beta_H = 0.3$, $\beta_L = 0.7$, $\bar{l} = 16$, $G_H = 5$, $G_L = 3$, $\theta_H = 4$, $\theta_L = 6$, and $\alpha = 0.6$.
- Utility function is unit and the principal and the agent earn a utility exactly equal to their profit.

After simplifying the model mentioned in Section 3.2 while taking the above into consideration, we can develop a new model for programming:

$$\max_{w_i, l_i} \quad \beta_H(P l_H^{\alpha} \theta_H^{\alpha-1} - w_H l_H) + \beta_L(P l_L^{\alpha} \theta_L^{\alpha-1} - w_L l_L)$$

Subject to:

$$W_H + G_L l_L - \theta_L q_L \geq 0$$
$$W_H + G_H l_H - \theta_H q_H \geq 0$$
$$W_L + G_L l_L - \theta_L q_L \geq W_H + G_L l_H - \theta_L q_H$$
$$W_H + G_H l_H - \theta_H q_H \geq W_L + G_H l_L - \theta_H q_L$$

The result of optimization problems will be:

Local optimal solution found
Objective value: 52.45122
Infeasibilities: 0.8881784E-15
Extended solver steps: 0
Total solver iterations: 30

Variable	Value	Reduced Cost
BETAH	0.3000000	0.000000
P	100.0000	0.000000
LH	1.632874	0.000000
ALPHA	0.6000000	0.000000
THETAH	4.000000	0.000000
WL	11.24761	0.000000
BETAL	0.7000000	0.000000
LL	2.111125	0.000000
THETAL	6.000000	0.000000
WH	12.64525	0.000000
GL	3.000000	0.000000
GH	5.000000	0.000000
Q	1000.000	0.000000

As it can be observed, the optimal contracts are $(W_H^* = 12.64, l_H^* = 1.63)$ and $(W_L^* = 11.24, l_L^* = 2.11)$ which provide participation and incentive compatibility constraints, simultaneously and provide a maximum utility equal to 52.45 for the employer (principal). These contracts can also well adapt to the theoretical foundations of the developed model.

5. Conclusion

In this study, lease and hire contracts in the Islamic setting of Iran was examined and asymmetric information problems associated with these contracts were analyzed. First, library method was used to examine the features of lease and hire contracts, experimental characteristics revealed in some previous studies, in Iran. Then, different aspects of asymmetric information, namely hidden information and hidden action, were mathematically modeled using contract theory. Regarding lease contract, first, optimal contract under complete information was modeled and then assumption of complete information was abandoned and incentive compatibility constraints were added to solve asymmetric information problems. In the next step, the assumption that "security deposit is more than verifiable damage" was abandoned and the possibility of overcoming verifiable damage to security deposit, which leads to lessor' loss, was examined. In the fourth step, financial constraints were added to the model and, finally, in the fifth step, the relaxing assumption of "a perfectly competitive market" was intuitively described.

We analyzed three possible asymmetric information problems in hire contract. They were:

- Adverse selection as the principal problem
- Adverse selection as the agent problem
- Moral hazard as the principal problem

Then mathematically model every one of them. The resultant model indicated a list of optimal lease and hire contracts in transactions which solve such asymmetric information problems as adverse selection and moral hazard through removing participation and incentive compatibility constraints.

Finally, the optimal contract was determined using hypothetical parameters and through LINGO software. As the results revealed, the main models provided for every transaction were solvable and the optimal contracts were obtainable.

Based on the results of this study, that the following suggestions can be offered:

- The government can determine the hypothetical parameters of this model, such as $\delta \; and \; P_H$, in order to find the optimal contracts for the Islamic-Iranian setting.
- The Ministry of Road and Urban Development can educate real estate agents about modern methodology of contracting, such as the model discussed in the present study, to solve asymmetric information problems.
- The Ministry of Labor and Social Affairs can enact labor and wage laws considering the present study's resultant model to motivate labors to reveal their private information.

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