# An Optimal Tax Relief Policy with Aligning Markov Chain and Dynamic Programming Approach

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## Abstract

In this paper, Markov chain and dynamic programming were used to represent a suitable pattern for tax relief and tax evasion decrease based on tax earnings in Iran from 2005 to 2009. Results, by applying this model, showed that tax evasion were 6714 billion Rials**. With 4% relief to tax payers and by calculating present value of the received tax, it was reduced to 3108 billion Rials. With regard to transition periods of tax-receipt and its discount index, the present value of future tax cash flows, during the future period, were 16034 billion Rials. Also, by means of dynamic programming, the break–even-point of tax relief index was determined. Consequently, if the discount rate were less than 22%, the optimal policy to increase tax revenue would be not to give discount and if it were more than 22%, the favorable policy would be to offer discount to tax payers.

## 1. Introduction

Many complexities and changes created in phenomena in the last decade led to an uncertainty and unpredictable behavior. These changes in economic and social systems are more than other systems. Markov Chain is a special case of random possible processes in which the condition of the system is just dependent on the last previous state. This model is able to calculate transition probabilities from one stage to other stages.

Reviews of received taxes show that parts of taxes are not gathered constantly in the due time and some are received with delay. First, it shows that the process of tax receipt is with a degree of probability. Second, part of receiving tax lasts to the next year during the transition period. So, the result would be an increase in tax evasion. Obviously, with these conditions, choosing an appropriate tax- policy can increase the reception rate in the due time and reduce the tax evasion.

For this purpose, with respect to the Markovian transition tax process and Dynamic Programming approach, a good model for decision-making in tax relief is introduced in four stages. In the first step, with regard to processes of receivable tax in 2005-2009, the algorithm transition was determined based on Markov chains. In the second step, steady state received tax and fundamental matrix were calculated. In the third step, receivable tax was calculated in the no relief policy, and relief policy. The results of the two states were compared. In the fourth step, by dynamic programming and relief policy, tax relief index was determined and optimal decision was taken.

## 2. Literature Review

Markov chain method has had numerous applications in various sectors such as agriculture, climate change, medicine and engineering. In the recent decade, the uses of Markov chain in the social and economic systems, especially in financial sectors, have found a suitable situation.

Brown (1965) used dynamic programming and transition probabilities for optimal decision making in tax relief in the US banks. This method decreased tax evasion compared to previous methods.

Engle and Hines (1999) have built a model to simulate and test long term tax evasion based on dynamic programming in the US. In their model, a taxpayer's current evasion is a decreasing function of prior evasion. They find the cross-section of evasion rates converges to a steady state and aggregate tax evasion approaches a limit even though

individual rates cycle.

Plassman and Tideman (2000) used Markov chain and Monte Carlo simulations to estimate the parameters in determining the effective tax rates for housing construction in Pennsylvania, USA in 1972 to 1994. The results showed that using this method to determine tax relief rate in future periods provided more useful information than other possible ways for decision-making.

Another research related to tax evasion subject and Markov chain was by Hanousek (2002). The main focus of the study was on estimating transition probabilities between the state of tax evading and not evading. In this research, non-panel surveys were conducted in 2000, 2002, and 2004. He used a dataset of 1062 individuals from the Czech Republic to forecast the evolution of tax evasion in that country. Respondents were asked whether they evaded never, sometimes, or frequently. This information was collected to classify individuals as evaders or non-evaders. It was concluded that the estimate of the shift allowed guessing how people might move between the categories of tax evading and not evading in the next five years and a small decrease in tax evasion would be seen then.

Derang & Cheng (2003) used Markov chain method to determine the pattern of facilities for credit institutions to customers in China. The application of Markov chain resulted in providing information for managers of this organization to determine policy guidelines to payment loans and to determine an appropriate credit policy to customers in the next periods.

Piunovskiy(2006) proposed a suitable pattern to analyze and decide appraisal with aligned dynamic programming and Markovian chain. This method applies in many countries for tax relief, special in East European countries.

Hanousek, Palda and Easter (2007), believed tax evasion is a prevalent problem in most developing countries, especially compared to the West counties. For this purpose, they selected 6,667 firms in 27 nations in 2002 to 2005. They sought to reveal these aspects in their study: 1-the perceived share of income reported by firms for tax purposes, 2-gift payments to tax collectors, 3-the accurate income reported for taxes across the countries in the 2005. They detected a relation between tax evasion and providing service to these firms from governments. The result of research showed the strong support for the

claim that the quality of government services and the level of corruption matter mightily. Clearly, other factors also matter, including the size and ownership of the firm, the fairness of the legal system, the level of competition, the tax rate, the quality of the bureaucracy, and the expectation of audits.

Saibeni (2010) used Markov chain method to predict receivable accounts in American companies. He compared different methods and concluded that Markov chain method was more useful to predict the process of collecting the receivable accounts than the other ones. Unlike Markov chain, in other methods, the transition periods of receivable accounts were not taken into consideration.

These studies represent the efficiency of Markov chain and dynamic programming in diverse contexts. This study attempted to work on this aspect in order to facilitate the way to increase tax receipt.

### 3. Methodology

### 3.1 Markov Chain Process

Markov chain is a special case of probability model. In this model, the current system state is only dependent on the last level. If a set of Markov processes is considered as the following, it is at any moment, one of the distinctive modes of S1……S(n). State of the system in discrete times and regular distances changes with a set of probabilities. If the state is shown as (qt) for (t=1, 2…n) in (t) time, to express the performance of this process in the form of Markov chain, It was needed to know the current state according to previous cases that is shown in the relation below:

P(qt=Sj)

$$P(q_t = S_j \mid q_{t-1} = S_i, q_{t-2} = S_k, ...)$$

$$P(q_t = S_j \mid q_{t-1} = S_i)$$

(1)

In this model:
P (qt) represents the system state.
P (qt = Sj | qt-1) represents the conditional probabilities transition.

Accordingly, the future behavior of the system only depends on the current state of (j) and it is not dependent on the previous behavior. So, to form a system with Markov model, system state and transition probabilities must be specified after determining the status of the model.

### 3.1.1 System State

System state specifies its position in a time period, such as, paid or unpaid taxes. For example, if the system time is zero, the mode is shown as below:

P (0) = [P1 (0), P2 (0), P3 (0)]                              (2)

In this relation:

P (0) = vector quantities Pi (0) for different modes.

P1 (0) = the probability that the system in zero time be in state1.

P2 (0) = the probability that the system in zero time be in state 2.

P3 (0) = the probability that the system in zero time be in state 3.

So, if it is assumed that the system at zero time is in state 1. Then, P (0) = [1, 0, 0]. Likewise, if the system in zero time is in state 2, then, P (0)=[0,1,0] or if the system is in state 3, then, it will be P(0)=[0,0,1]. With the use of these conditions, in Markov processes, the transition states can be defined.

Since, in Markov process, the system state at any time is dependent on the previous mode, probability transitions with regard to (n) periods is defined as below:

$p(n)=p(0)\ p^n$ =[p1(n),p2(n),...pi(n)]=[p1(0),p2(0),...pm(0)]     (3)

P(n)= Vector mode in (n) time

P(0)= Vector mode in (0) time

n= Number of time periods

0 =The initial condition

m= Number of possible states

### 3.1.2 Transition Probabilities

Transition probabilities represent the move from a state to the other state during a specified period. These changes are related to the current state. For example, the probability that in the previous state, tax had not been paid, and in this period, would be paid is related to the previous state. It is shown as a conditional probability.

Pij = Pr {X1 = j | X0 = i}                              (4)

According to space state of S = {0, 1, 0}

$\sum j\ pij = 1\ \ \forall\ i$    and   $pij\ \geq 0\ \ \forall\ i,$

If possible probabilities are considered as P (ij), in line with conditions and probabilities in Markov chain, the transition matrix is defined as:

$$
Pij=
\begin{array}{c}
\phantom{Pij=}\\
1\\
2\\
3\\
\cdot\\
\cdot\\
\cdot\\
i
\end{array}
\begin{array}{cccc}
1 & 2 & 3\ldots & \ldots\ldots j\\
\left[\begin{array}{cccc}
P_{11} & P_{12} & P_{13}\ldots\ldots.P_{1j}\\
P_{21} & P_{22} & P_{23}\ldots\ldots P_{2j}\\
P_{31} & P_{32} & P_{33}\ldots\ldots P_{3j}\\
\ldots & \ldots. & \ldots. & \ldots.\\
\ldots & \ldots. & \ldots. & \ldots.\\
\ldots & \ldots. & \ldots. & \ldots.\\
P_{i1} & P_{i2} & P_{i3} & P_{ij}
\end{array}\right]
\end{array}
\qquad (5)
$$

According to this matrix, transition probabilities are moving from mode 1 to (i) to the one of the states of 1 to (j). In this model, for example, the probability that the system is in state 1 and transits to state 1 is shown with P(11). The probability that the system is in (i) state and transits to state (j) is shown with P(ij). If the probability of changes is considered between 6 modes, it can be said that probability that the system be in zero state and would stay in it is zero. The probability that the system be in zero state and transits in state 2 is equal to 0.5 Eventually, the probability that the system be in state 2 and transfer to the zero mode will be equal to 1. Since, in this matrix, the transition probability is only possible from state 2 to zero state, it is called "absorbing state"(state 1, 4, 5, and 6). ( Maskin and Taylor, 2001). (Figure 1).

$$
P =
\begin{array}{c}
[0\ 1\ 1]\\
[0\ 2\ 0]\\
[0\ 0\ 2]\\
[2\ 0\ 0]\\
[1\ 1\ 0]\\
[1\ 0\ 1]
\end{array}
\begin{array}{c}
[0\ 1\ 1]\ \ [0\ 2\ 0]\ \ [0\ 0\ 2]\ \ [2\ 0\ 0]\ \ [1\ 1\ 0]\ \ [1\ 0\ 1]\\
\left[\begin{array}{cccccc}
0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2}\\
\frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{2}\\
\frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0
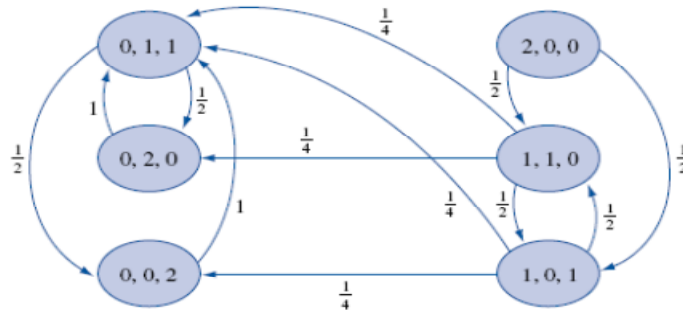\end{array}\right]
\end{array}
$$

Figure 1. Transition processes between state 0, 1 and 2

### 3.2 Dynamic Programming Method

Dynamic programming is one of the best operation research methods for solving complex problems by breaking them down into a sequence of decision steps. This is done by defining a sequence of value functions $f_1$, $f_2$, ..., $f_n$, with an argument $S$ representing the state of the system at times $i$ from 1 to $n$. The definition of $f_n(S)$ is the value obtained in state $S$ at the last time $n$. The values $f_i$ at earlier times $i = n - 1$, $n - 2$, ..., 2, 1 can be found by working backwards, using a recursive equation. For $i = 2$, ..., $n$, $f_{i-1}$ at any state $S$ is calculated from $f_i$ by maximizing a simple function of the gain from decision $i - 1$ and the function $f_i$ at the new state of the system if this decision is made. Since $f_i$ has already been calculated for the needed states, the above operation yields $f_{i-1}$ for those states. Finally, $f_1$ at the initial state of the system is the value of the optimal solution. optimal decision in this stage was calculated using the following formula:

$$f_n(i) = \max_k \left\{ f_n (s, v_n) \right\} \qquad (6)$$

Recursive equation determined such that:

$$f_n(i) = \max_k \left\{ v_i^k + f_{n+1}^{s}(i) \right\} \qquad (7)$$

$f_n(i)$ = The optimal decision in stage n.

$V_i^k$ = the value of optimal decision in stage (n-1).

### 3.3 Aligning dynamic programming with Markovian chain

With regard to Markovian chain, suppose $P^s$ is the transition matrix associated with the $S$ stationary policies, S=1, 2, 3…S. $V_i^s$ is expected revenue of policy $S$ from state i, i=1,2,..*m*. Based on the above relation, expected revenue of policy $S$ per transition step is calculated with this formula:

$$E^S = \sum_{i=1}^{m} p_i^s v_i^s \tag{8}$$

The optimum policy for $s^*$ is determined with:

$$E^{S*} = Max\{E^S\} \tag{9}$$

Optimal policy for each stage includes the optimization of the current stage and the previous stage. Therefore, the recursive function, in this model, includes the optimization of the current stage and the prior stage. If instead of $E^S$ apply $f_n(i)$ and *k*=stage, then value determination each step is calculated:

$$f_n(i) = \max_k \left\{ v_i^k + \sum_{j=1}^{m} p_{ij}^k f_{n+1}^s(j) \right\} \tag{10}$$

In this equation:

$f_n(i)$ = The optimal expected revenue of stage n.

$V_i^k$ = optimal decision in stage (n-1).

$\sum_{j=1}^{m} p_{il}^k f_{n+1}(j)$ = Optimal decision value in stage (n) according to

conditions of possible tax receipt.

To calculate the present received value, by use of discount factor (0< α <1), the recursive equation can be written as:

$$f_n(i) = Max \left\{ V_i^k + \alpha \sum_{j=1}^{m} p_{ij}^k f_{n+1}(j) \right\} \tag{11}$$

The main objective model is to optimize the above equation.

## 3.4 Application of Markov Chain and dynamic programming for Tax Relief Pattern

One of the major challenges of tax receipt, in our country, is non-complete receipt of anticipated tax. On the other hand, part of tax in each period is not received in the due time. This issue causes the increasing risk of tax receipt and it will affect the value of future received payments. Considering features of Markov chain method, in this model, "state" represents the present tax receipt situation. Transition probabilities represent time process of tax receipt in different parts. (Yaniv,1994).

Tax receipt in previous periods is shown based on Markov chain in the following matrix. This matrix has two specific "conditions" and 12 probable "states". Specified conditions include a condition that tax is certainly received in the due time. This case is shown with R. The condition which it is not certainly received is shown with E. Other possible conditions which are shown as 1 to12 mean the tax receipt during transition period of a 12-month in a year. Accordingly, if tax is received one month later, it is shown with 1 and if it is received after 12 months, it is shown with 12. Finally, if the tax is not received until the end of a 12- month period, it is considered as non-received. In this matrix, for example, the case that R is equal to 1 indicates the probability that the tax has already been received and this month is also going to be received. The number 0.45 in the third row indicates that if three months are passed and the tax is not fully received yet, 0.45 is probable that it is going to be received in later periods. 0.35 in the second month. 0.15 in the seventh month. Finally, 0.05 that this tax is not probably received at all (this state is shown with E).

$$
\text{Pij}= \begin{array}{c} R \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ E \end{array}
\begin{pmatrix}
\begin{array}{ccccccccccccccc}
R & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & E \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
.56 & .30 & 0 & .1 & 0 & 0 & .04 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
.45 & .25 & 0 & 0 & 0 & 0 & 0 & .20 & 0 & 0 & 0 & .1 & 0 & 0 \\
.45 & .35 & 0 & 0 & 0 & 0 & 0 & .15 & 0 & 0 & 0 & .1 & 0 & .05 \\
.62 & .21 & .15 & 0 & 0 & .03 & 0 & .20 & 0 & 0 & 0 & 0 & 0 \\
.46 & .28 & .18 & 0 & 0 & .05 & 0 & 0 & 0 & 0 & 0 & 0 & .02 \\
.66 & .22 & .12 & 0 & 0 & .0 & 0 & 0 & 0 & 0 & 0 & 0 & .0 \\
.40 & .25 & .3 & 0 & 0 & .05 & 0 & 0 & 0 & 0 & 0 & 0 & .05 \\
.55 & .20 & .10 & 0 & 0 & .05 & 0 & 0 & 0 & 0 & 0 & 0 & .1 \\
.3 & .28 & .12 & 0 & 0 & .15 & 0 & 0 & .25 & 0 & 0 & 0 & .0 \\
.75 & .20 & .0 & 0 & 0 & .05 & 0 & 0 & 0 & 0 & 0 & 0 & .0 & 0 \\
.5 & .0 & .3 & 0 & 0 & .15 & 0 & 0 & 0 & 0 & 0 & 0 & .05 & 0 & 0 \\
.21 & .35 & .4 & .0 & 0 & .04 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}
\end{pmatrix}
$$

To calculate, according to tax receipt transition matrix based on Markov chain, this matrix is divided into four separate sections as follows:

$$
\text{Pij} = \begin{array}{c|c} I & O \\ \hline R & Q \end{array} \tag{12}
$$

In this matrix, I= unit matrix, O= zero matrix, R= matrix of transition probabilities which in next periods reaches to the absorbing state and Q= matrix of transition probabilities between all states of non- absorb.

Considering the defined matrix, the basic step to identify the behavior of this system is to compute the fundamental matrix. The fundamental matrix is calculated as below:

$$
F= (I-Q)^{-1} \tag{13}
$$

In this matrix, for example, the first month shows that if tax receipt transfers to the first month, 10% is likely that this tax is going to be received in the first month, 20% in the second month, 10% in the fourth month, 14% in the seventh month and 5% in the ninth month.

$$F = \begin{bmatrix}
.1 & .2 & 0.1 & 0 & 0.14 & 0 & .05 & 0 \\
.0 & .0 & 0 & .4 & 0 & 0 & 0 & 0 & 0 & 0 \\
.05 & .2 & 0 & 0 & 0 & 0 & 0 & .05 & 0 & 0 \\
.0 & .1 & .0 & 0 & 0 & .03 & 0 & 0 & 0 & 0 \\
.0 & .2 & .0 & 0 & 0 & .05 & 0 & 0 & 0 & 0 \\
.05 & .2 & .10 & 0 & 0 & .0 & 0 & .05 & 0 \\
.0 & .25 & .4 & 0 & 0 & .05 & 0 & 0 & .1 & 0 \\
.05 & .0 & .0 & 0 & 0 & .05 & .12 & 0 & 0 \\
.1 & .0 & .10 & 0 & 0 & .1 & 0 & 0 & 0 & 0 \\
0 & .2 & .0 & 0 & 0 & .2 & 0 & 0 & 0 & .01 \\
\end{bmatrix}$$

As the ultimate goal of this process is to reach a steady state, a fundamental matrix leads to a steady state. Considering the state of tax receipt transition matrix, the data were analyzed through QM software and were simulated in five year period. The results show the extent of tax receipt in future periods.

The following matrix shows the tax receipt in 13 state (1, 2, and 3…E) and five-year period (2005 - 2009). For example, in the first month in 2005, with the probability of 0.18, 3309 billion Rials were received and with 0.04 probabilities, 809 billion Rials would be non-receivable. Other tax receipt information during the transitional period is shown too.

| Month Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2005 | .18 | .14 | .12 | .1 | .11 | .09 | .08 | .05 | .02 | .04 | .03 | .01 | .04 |
| 2006 | .17 | .15 | .13 | .11 | .09 | .08 | .07 | .06 | .03 | .05 | .01 | .02 | .035 |
| 2007 | | .21 | .12 | .12 | .08 | .09 | .06 | .06 | .05 | .03 | .01 | .02 | .03.04 |
| 2008 | 13 | .18 | .17 | .12 | .1 | .09 | .08 | .06 | .09 | .01 | .02 | .04. | .17 |
| 2009 | .15 | .16 | .14 | .14 | .13 | .09 | .08 | .07 | .05 | .08 | .07 | .04 | .04 |

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | E | Total Tax Received |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2005 | 3309 | 2757 | 2573 | 2206 | 2206 | 1814 | 919 | 551 | 735 | 551 | 184 | 551 | -809 | 17549 |
| 2006 | 3816 | 3367 | 2918 | 2245 | 2020 | 1796 | 1571 | 1347 | 673 | 1010 | 224 | 449 | -1010 | 22449 |
| 2007 | 5750 | 5476 | 3286 | 2190 | 2190 | 1643 | 1643 | 1369 | 821 | 274 | 548 | 821 | -1369 | 27381 |
| 2008 | 6296 | 5965 | 5302 | 3314 | 3314 | 1988 | 1823 | 994 | 1011 | 331 | 646 | 663 | -1491 | 33137 |
| 2009 | 7631 | 7122 | 7122 | 6105 | 5087 | 3561 | 3052 | 2035 | 2035 | 2544 | 1526 | 1017 | -2035 | 50872 |
| Total | | | | | | | | | | | | | -6714 | |

According to tax relief policy, to determine the index of tax relief, this pattern is reviewed again. With tax relief policy, tax receipt increases. As tax payers will benefit from tax relief, part of tax will not be received from them. For example, with tax relief in the first month, the probability of earning revenue increases from18% to 22% and the earned revenue increases from3309 billion Rials to 4165 billion Rials. Also in the whole examined period, the probabilities of not earning revenue decreases from4% to 1% and the earning revenue decreases from 809 billion Rials to 379 billion Rials.

$$
\begin{array}{c}
\begin{array}{ccccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & E
\end{array} \\
\begin{array}{c}
2005 \\
2006 \\
2007 \\
2008 \\
2009
\end{array}
\left[
\begin{array}{ccccccccccccc}
.22 & .16 & .15 & .12 & .12 & .09 & .11 & .05 & .02 & .02 & .01 & .01 & .01 \\
.21 & .17 & .15 & .13 & .11 & .09 & .05 & .03 & .02 & .02 & .01 & .01 & .00 \\
.22 & .21 & .18 & .12 & .09 & .07 & .04 & .03 & .01 & .01 & .01 & .01 & .01 \\
.20 & .19 & .18 & .12 & .08 & .05 & .05 & .04 & .03 & .01 & .02 & .01 & .01 \\
.18 & .17 & .15 & .13 & .10 & .09 & .05 & .04 & .03 & .02 & .01 & .02 & .01
\end{array}
\right]
\end{array}
$$

| year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | E | Total tax received |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 2005 | 4165 | 3029 | 2840 | 2272 | 1704 | 1325 | 1136 | 947 | 379 | 379 | 189 | 189 | -379 | 18933 |
| 2006 | 4856 | 3931 | 3468 | 2775 | 2543 | 1850 | 1156 | 694 | 462 | 462 | 231 | 231 | -486 | 23146 |
| 2007 | 6205 | 5923 | 5076 | 3102 | 2256 | 1974 | 1128 | 846 | 282 | 282 | 282 | 282 | -564 | 28202 |
| 2008 | 6826 | 6485 | 6144 | 4096 | 2730 | 1707 | 1707 | 1365 | 1024 | 341 | 683 | 341 | -683 | 34131 |
| 2009 | 9432 | 8908 | 7860 | 6812 | 5240 | 4192 | 2620 | 2096 | 1572 | 1048 | 524 | 1048 | -996 | 52346 |
| Total | | | | | | | | | | | | | | -3108 |

Results of taking this tax policy, based on tax relief and non-relief index during 2005-2009 are shown in table 1. For example, based on the data in 2005, government can provide a discount of 1384 billion Rials to taxpayers. In this case, the relief-index will reach to break-even-point. For the other years, tax-relief index is shown in table 1.

Table 1: Comparison of tax receipt in the state of tax relief and non-tax relief policy - Billion Rials

| Year | Tax receipt in the state of non–tax relief policy | Tax receipt in the state of tax-relief policy | Maximum tax-relief Index | Compound interest factor | Future value |
|------|------|------|------|------|------|
| 2005 | 17549 | 18933 | 1384 | 1.22 | 1661 |
| 2006 | 22449 | 23146 | 697 | 1.23 | 857 |
| 2007 | 27381 | 28202 | 821 | 1.28 | 1051 |
| 2008 | 33137 | 34131 | 994 | 1.37 | 1362 |
| 2009 | 50872 | 52346 | 1474 | 1.2 | 1769 |
| Total | 151388 | 156758 | 5370 | _ | 6700 |

As during the transition period tax is received continuously in 12

months in each year; present value of tax will be different in the transition. For this purpose, the Dynamic Programming method was used to calculate the present value of the received tax and to appraise an optimal policy. The index used to calculate the present value of the tax is the real inflation rate including nominal inflation and interest rate. This index is applied to select optimal policy in dynamic programming. In this method, optimal policy, for each stage, includes the optimization of the current stage and the previous stage. Therefore, the recursive function, in this model, includes the optimization of the current stage and the prior stage. The optimal decision is calculated according to the following equation:

$$f_n(i) = \text{Max} \left\{ V_i^k + \alpha \sum_{j=1}^{m} p_{ij}^k f_{n+1}(j) \right\}$$

In this equation:

$f_n(i)$ = The optimal decision in stage (n) (each year as a stage of decision-making)

$V_i^k$ = optimal decision in stage (n-1).

$\alpha$ = Rate of discount factor (real inflation rate).

$\sum_{j=1}^{m} p_{il}^k f_{n+1}(j)$ = Optimal decision value in stage (n) according to conditions of possible tax receipt

By using the above function, the value of received tax in 2005 - 2009, based on non-relief tax policy is calculated as the following. These results determine the selection of an optimal policy in each year.

**f(2005)**={.22f(1)+.16f(2)+.15f(3)+.12f(4)+.12f(5)+.09f(6)+.11f(7)+.05f(8)+.02f(9)+ .02f(10)+.01f(11)+.01f(12)-.01f(13)}=**17159**
**f(2006)**=17159+{.21f(1)+.17f(2)+.15f(3)+.13f(4)+.11f(5)+.09f(6)+.05f(7)+.03f(8)+ .02f(9) + .02f(10)+.01f(11)+.01f(12)-.00f(13)}=**19893**
**f(2007)**=19893+{.22f(1)+.21f(2)+.18f(3)+.12f(4)+.09f(5)+.07f(6)+.04f(7)+.03f(8)+ .01f(9)+ .01f(10)+.01f(11)+.01f(12)-.01f(13)}=**23985**
**f(2008)**=23985+{.20f(1)+.19f(2)+.18f(3)+.12f(4)+.08f(5)+.05f(6)+.05f(7)+.04f(8)+ .03f(9)+ .02f(10)+.01f(11)+.01f(12)+.01f(13)}=**28678**
**f(2009)**=28678+{.18f(1)+.17f(2)+.15f(3)+.13f(4)+.1f(5)+.09f(6)+.05f(7) + .04f(8)+.03f(9)+ .02f(10)+.01f(11)+.02f(12)-.01f(13)}=**45557**

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | E | Total tax received |
|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|--------|
| 2005 | 3256 | 2871 | 2553 | 2069 | 2037 | 1649 | 828 | 485 | 637 | 470 | 154 | 455 | -668 | 17159 |
| 2006 | 3751 | 3254 | 2772 | 2096 | 1854 | 1620 | 1393 | 1174 | 577 | 851 | 186 | 365 | -822 | 19893 |
| 2007 | 5633 | 5255 | 3088 | 2017 | 1976 | 1452 | 1422 | 1161 | 682 | 223 | 436 | 641 | -1069 | 23985 |
| 2008 | 6136 | 5665 | 4908 | 2989 | 2913 | 1704 | 1522 | 809 | 802 | 256 | 487 | 487 | -1095 | 28678 |
| 2009 | 7515 | 6907 | 6802 | 5741 | 4712 | 3248 | 2742 | 1800 | 1773 | 2182 | 1289 | 846 | -1693 | 45557 |

In addition, value of received tax, based on the tax relief policy, is as the following:

**f(2005)**={.18f(1)+.14f(2)+.12f(3)+.1f(4)+.11f(5)+.09f(6)+.08f(7)+.05f(8)+.02f(9)+ .04f(10)+.03f(11)+.01f(12)+.01f(13)}=**17465**

**f(2006)**=17465+{.17f(1)+.15f(2)+.13f(3)+.11f(4)+.09f(5)+.08f(6)+.07f(7)+.06f(8)+ .03f(9) + .05f(10)+.01f(11)+.02f(12)+.035f(13)}=**21253**

**f(2007)**=21253+{.21f(1)+.2f(2)+.12f(3)+.08f(4)+.09f(5)+.06f(6)+.05f(7)+.03f(8)+ .01f(9)+ .02f(10)+.03f(11)+.01f(12)+.04f(13)}=**25770**

**f(2008))**=25770+{.19f(1)+.18f(2)+.17f(3)+.12f(4)+.1f(5)+.09f(6)+.08f(7)+.06f(8)+ .09f(9)+ .01f(10)+.02f(11)+.04f(12)+.03f(13)}=**30434**

**f(2009)**=30434+{.151f(1)+.16f(2)+.14f(3)+.14f(4)+.13f(5)+.09f(6)+.08f(7)+.07f(8)+ .05f(9) + .08f(10)+.07f(11)+.04f(12)+.03f(13)}=**48306**

| year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | E | Total tax received |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2005 | 3899 | 2834 | 2701 | 2132 | 1573 | 1204 | 1016 | 833 | 328 | 323 | 159 | 156 | -313 | 17465 |
| 2006 | 4773 | 3798 | 3294 | 2590 | 2334 | 1669 | 1025 | 605 | 396 | 389 | 191 | 188 | -395 | 21253 |
| 2007 | 6078 | 5683 | 4772 | 2856 | 2035 | 1744 | 976 | 717 | 234 | 229 | 225 | 220 | -440 | 25770 |
| 2008 | 6653 | 6159 | 5687 | 3695 | 2401 | 1462 | 1425 | 1111 | 812 | 264 | 514 | 251 | -501 | 30434 |
| 2009 | 9288 | 8639 | 7506 | 6407 | 4853 | 3823 | 2353 | 1854 | 1369 | 899 | 443 | 872 | -828 | 48306 |

Results of these two policies are shown in the table below:

Table 2: Comparison of received tax in state of tax relief and non-tax relief based on the discount factor- Billion Rials

| Year | Real inflation index (discount factor) % | Tax receipt in state of non–tax relief policy | Tax receipt in state of tax relief policy with compound interest factor | Maximum tax-relief Index | Optimal policy |
|---|---|---|---|---|---|
| 2005 | 21 | 17465 | 17159 | -306 | non –tax-relief policy |
| 2006 | 23 | 19893 | 21253 | 1360 | tax-relief policy |
| 2007 | 28 | 23985 | 25770 | 1785 | tax-relief policy |
| 2008 | 36 | 28687 | 30434 | 1756 | tax-relief policy |
| 2009 | 20 | 48758 | 48306 | -451 | non–tax-relief policy |

According to the information in table 2 and selection of a best policy, an optimal decision based on the discount factor, the present value of the received tax and the tax relief will be different. In 2005 and 2009, the discount factor (21%, 20%) was less than tax relief (22%), the tax discount policy was not appropriate. However, in 2006, 2007 and 2008, the discount factor rate was more than the tax relief. Founded on the results of calculations, the rate of 22% is considered as the break-even-point. If the rate of the discount-factor is less than 22%, it is better to apply the policy of non-discount payment. But if the discount-factor rate is more than 22%, taking policies of tax relief would be desirable.

## 4. Discussion and Recommendation

One of the important challenges of government in tax income is to increase receipt of anticipated tax and to decrease tax evasion. For this purpose, various incentive policies can be used to receive tax in the due time to increase tax income. Taxes are one of the important sources of income in the developed countries but in developing countries tax evasion is a prevalent problem (Hanousek, 2007).

Now, in Iran, appropriate policies are not used for tax relief. Moreover, the present policies are subjective with no consideration of scientific models. For this, the study of tax-payers' behaviors, tax discount rate in previous periods and their impacts on tax receipt can appropriately provide a model based on the present revenue value. In this paper, a desired pattern for future periods was presented with Markov chain attributes and dynamic programming in identifying characteristics of taxpayers' behaviors and transition periods of tax receipt.

One of the results of this study was to provide an appropriate pattern for tax relief according to Markov chain and the dynamic programming. According to the data of non-received taxes and transition periods of the tax receipt, it was observed that in the total examined period, 6714 billion Rials among the anticipated tax is not collected. The process of tax revenues has also a transitional period. With the transition period of tax receipt during 12 months and the discount factor, the present value of future (2010-2014), and tax cash flows will be equivalent to 16,034 billion Rials. In addition, calculated data can be as a basis for tax relief decision-making. According to findings of this study, firstly, it is recommended to use appropriate incentive policies in order to increase tax revenues compared to the tax relief. Secondly, the shorter the

transitional period of tax receipt, the more the present value of future cash flows and the higher value for the government. By applying this policy, it is expected to reduce tax evasion. Hanousek, J, Palda and Easter (2007), in their research concluded that near to 40% of tax income was not received in European East countries.

Another important result of this study is the use of Dynamic Programming method and the implementation of the discount factor in order to take an appropriate policy of tax relief against non-relief tax. Based on the results of this method, the discount factor rate of 22% is considered as the break-even-point. If the discount factor is less than 22%, the optimal policy is no relief. But, by increasing the discount factor, and by reducing value of money, the appropriate policy is tax relief. As in this state, the present value of cash flows increases in the transitional period. Piunovskiy (2006) in her research in Russian perceived that the tax relief index for firms was near to 7% and for personal income was 4%. But in Poland tax relief index is different.

**Appendix:**

Appendix 1: Input matrix for received tax in the transition period

State10 : State8    0

| From \ To | State1 | State2 | State3 | State4 | State5 | State6 | State7 | State8 | State9 | State10 | State11 | State12 | State13 | State14 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| State1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| State2 | 0.56 | 0.3 | 0 | 0.1 | 0 | 0 | 0.04 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| State3 | 0.45 | 0.25 | 0 | 0 | 0 | 0 | 0 | 0.2 | 0 | 0 | 0.1 | 0 | 0 | 0 |
| State4 | 0.45 | 0.35 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0 | 0 | 0.1 | 0 | 0.05 | 0 |
| State5 | 0.41 | 0.21 | 0.15 | 0 | 0 | 0.03 | 0 | 0.2 | 0 | 0 | 0 | 0 | 0 | 0 |
| State6 | 0.46 | 0.28 | 0.18 | 0 | .01 | 0.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0.02 | 0 |
| State7 | 0.54 | 0.22 | 0.14 | 0 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.08 | 0 |
| State8 | 0.4 | 0.2 | 0.3 | 0 | 0 | 0.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0 |
| State9 | 0.55 | 0.2 | 0.1 | 0 | 0 | 0.05 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| State10 | 0.4 | 0.28 | 0.12 | 0 | 0 | 0.15 | 0 | 0 | 0.05 | 0 | 0 | 0 | 0 | 0 |
| State11 | 0.35 | 0.26 | 0.21 | 0.01 | 0 | 0.08 | 0 | 0 | 0.01 | 0 | 0 | 0 | 0.08 | 0 |
| State12 | 0.33 | 0.23 | 0.18 | 0 | 0 | 0.15 | 0 | 0 | 0 | 0 | 0.05 | 0 | 0.06 | 0 |
| State13 | 0.32 | 0.27 | 0.2 | 0.12 | 0.04 | 0.03 | 0 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 |
| State14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Initial Prob. | 6.22 | 3.05 | 1.58 | 0.23 | 0.07 | 0.59 | 0.04 | 0.11 | 0.06 | | 0.25 | | 0.44 | 1 |
| State Cost | 7631 | 7122 | 7122 | 6105 | 5087 | 3561 | 3052 | 2035 | 2035 | 2544 | 1526 | 1017 | 986 | -2035 |

Appendix 2: Simulation process in transition period for received tax

| 06-26-2011 | Time Period | Total Expected Return/Cost |
|---|---|---|
| 1 | 1 | 7142.3290 |
| 2 | 2 | 7406.7100 |
| 3 | 3 | 7518.4370 |
| 4 | 4 | 7576.6360 |
| 5 | 5 | 7604.1140 |
| 6 | 6 | 7617.8240 |
| 7 | 7 | 7624.5070 |
| 8 | 8 | 7627.8080 |
| 9 | 9 | 7629.4280 |
| 10 | 10 | 7630.2270 |

Appendix 3: Transition matrix based on the first year

**0.046677**

| State | Initial State Probability | Resulted State Probability |
|---|---|---|
| State 1 | 0.492089 | 0.741258 |
| State 2 | 0.241297 | 0.142168 |
| State 3 | 0.125000 | 0.023877 |
| State 4 | 0.018196 | 0.028505 |
| State 5 | 0.005538 | 0.001922 |
| State 6 | 0.046677 | 0.005799 |
| State 7 | 0.003165 | 0.009652 |
| State 8 | 0.008703 | 0.027714 |
| State 9 | 0.004747 | 0.000198 |
| State 10 | 0 | 0 |

**The number of time periods from initial:**      **1**
**Expected cost or return:**      **7,142.330000**

Appendix 4: Transition matrix based on the total years and stationary state

| State | Initial State | Resulted State | Resulted State | Resulted State | Resulted State | Resulted State | Resulted State | Resulted State | Resulted State | Resulted State |
|---|---|---|---|---|---|---|---|---|---|---|
| State1 | 492089 | ,741258 | 0.8708 | 0.9360 | 0.9684 | 0.9844 | 0.9923 | 0.9962 | 0.9962 | 0.8708 |
| State2 | 241297 | ,142168 | 0.0733 | 0.0371 | 0.0184 | 0.0091 | 0.0045 | 0.0022 | 0.0022 | 0.0733 |
| State3 | 125000 | ,023877 | 0.0149 | 0.0055 | 0.0029 | 0.0013 | 0.0007 | 0.0003 | 0.0003 | 0.0149 |
| State4 | 018196 | ,028505 | 0.0149 | 0.0080 | 0.0040 | 0.0020 | 0.0010 | 0.0005 | 0.0005 | 0.0149 |
| State5 | 005538 | ,001922 | 0.0004 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0004 |
| State6 | 046677 | ,005799 | 0.0030 | 0.0011 | 0.0006 | 0.0003 | 0.0001 | 0.0001 | 0.0001 | 0.0030 |
| State7 | 003165 | ,009652 | 0.0057 | 0.0029 | 0.0015 | 0.0007 | 0.0004 | 0.0002 | 0.0002 | 0.0057 |
| State8 | 008703 | ,027714 | 0.0067 | 0.0039 | 0.0016 | 0.0008 | 0.0004 | 0.0002 | 0.0002 | 0.0067 |
| State9 | 004747 | ,000198 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |
| State10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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